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Preliminary Design of a Hollow Displacer for a Low Temperature Difference Stirling Engine

Zachary de Rouyan[1], Connor Speer[1], David S. Nobes[1*]

[1]Mechanical Engineering, University of Alberta, Edmonton, Canada

*dnobes@ualberta.ca

Abstract – This paper investigates six hollow displacer designs numerically and with the help of finite element analysis software. The displacer are being developed for use in a pressurized low temperature differential Stirling Engine (LTDSE) currently under development as a modelling software validation tool that uses a 95°C hot source and a 5°C cold source with aim of producing above 100 W. Current software for Stirling Engines has focused on high temperature differential Stirling Engines (HTDSE), and so a new software that more accurately simulates LTDSE's could be useful in the design of Stirling Engines meant to take advantage of low temperature applications such as low temperature geothermal sources. Due to the low temperatures inherent in a LTDSE, thermal resistivity of the displacer piston is not a concern, so the six designs are compared based on their ability to withstand the pressure swings while being light. It was found that an AISI 304 hemisphere design, at 10.46 kg, was the best design for the engine being developed.

I. Introduction

Stirling engines are closed-cycle heat engines that rely on heat transfer from an external source rather than internal combustion to generate power [1]. They have few moving parts, are relatively quiet, and can use a variety of heat sources. Stirling engines have been developed for mobile electric power, submarine engines, waste heat recovery, cryocooling, space and geothermal power generation, and micro-scale combined heat and power applications [2]. Stirling engines have been built for a wide range of heat source temperatures, including down to a few degrees above ambient [1].

The function of the displacer in a Stirling engine is to displace the working fluid back and forth through a series of heat exchangers which separate a hot expansion space and a cold compression space. The displacer does not change the engine volume and is meant to separate the compression and expansion spaces while facilitating the mass transfer between the two. The displacer must withstand the pressure fluctuations inherent to the Stirling cycle, while being light to mitigate inertial forces. The potentially high temperature gradient, especially in high temperature differential Stirling Engines (HTDSE) with upwards of 150°C temperature difference between hot and cold source, means that the displacer must also be thermally resistive. To get a similar power out as a HTDSE, a low temperature difference Stirling engine (LTDSE) must have a combination of larger working fluid volume, higher fill pressure, or the pressure the engine is filled to before running, and larger heat exchanger surface area [3]. The first two elements of this list affect the displacer design. The larger working fluid volume means that the displacer will have a combination of higher diameter and higher stroke. Despite the larger diameter, the displacer will also have to resist larger pressures if a LTDSE were to be designed to have a similar power. The conflict between the need for a larger displacer that resists higher pressures while remaining light makes designing a displacer for a LTDSE challenging.

The engine that this displacer is being designed for will utilize a 95°C hot source and a 5°C cold source that simulates what might be achieved with a common LTDSE like in [4]. It is being designed to withstand a ten-atmosphere fill pressure with variable power piston strokes so that variable compression ratios can be tested. So, the displacer for this engine will be subject to a wide range of pressures.

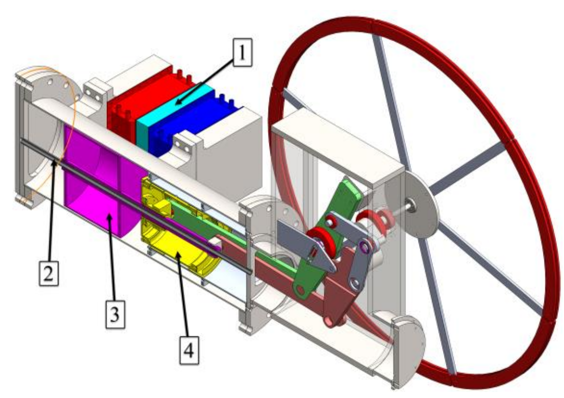

Figure 1: Preliminary engine model with the heat exchanger assembly (1), guide rod (2), displacer piston (3), and power piston (4) shown

Previous displacer designs for LTDSE have either been solid or used a sheet metal design with a flat face [4]–[6]. However, the larger scale of the new engine compared with [4] and [6] means that a flat face would likely produce a displacer that is too heavy to be used in the new engine. The larger scale also means that a solid displacer would be too heavy.

This paper investigates design considerations for the displacer of a LTDSE. Since this displacer is being designed for a LTDSE, thermally resistivity is less of a concern. So, this paper focuses on comparing the designs by weight and magnitude of the factor of safety at the expected maximum operating pressures. While many designs were considered, six designs that proved to be promising will be presented in this paper. They will be compared numerically and with the help of finite element analysis (FEA).

II. Comparison with Previous Designs

Hollow displacer designs have been developed previously as described in [5] and [4]. However, the displacer in [5] was for a rotary LTDSE and the displacer in [4] was for a high temperature differential Stirling engine that used between 145-242°C hot source temperature and a 21°C cold source temperature. However, both were designed with a ten-atmosphere maximum fill pressure in mind. Despite the high pressures, both displacers were designed with flat metal faces on either side of the displacer. However, the rotary displacer was able to incorporate internal supports. This new displacer design will need to be easily disassembled so it can be replaced or modified if different displacer geometry is to be tested in the future. This limits the ability to use internal supports. The high temperature differential Stirling engine displacer in [4] was ~100 mm in diameter. While being close in geometry to what could be incorporated into this new engine, the 355.6 mm (14 in) diameter makes it difficult to withstand the expected pressure swings compared with [4].

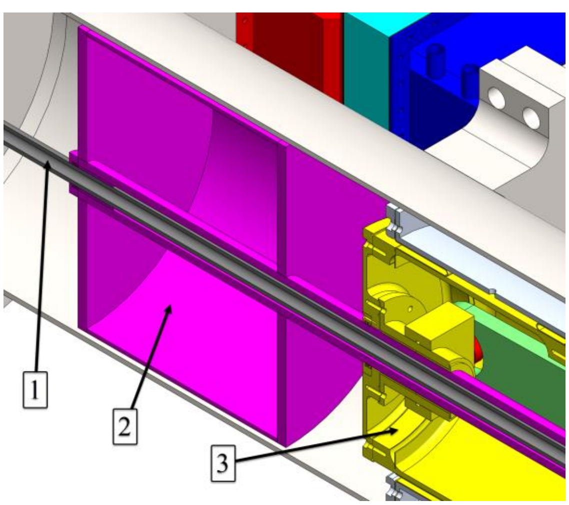

Figure 2: Close up view of the guide rod (1), displacer (2), and power piston assembly (3)

Unlike the previously mentioned displacer designs, this engine uses a central guide rod to constrain the linear motion of the displacer, rather than a ring around the outside of the displacer. Unlike [4], the LTDSE being developed will have the displacer set horizontally due to size constraints of the room. This puts pressure on the seals in the power piston and displacer. Since the seals are a common failure point for Stirling engines, it was determined that the added complexity of a guide rod through both power piston and displacer did not outweigh the benefits of its addition to the engine. To allow for the associated linear bearings, the internal rod that will be attached to the displacer is assumed to have an outer diameter of 50.8 mm (two inches) and an inner diameter of 38.1 mm (1.5 in) to ride on a 25.4 mm (one inch) guide rod.

Figure 1 and Figure 2 show the displacer in the context of the overall engine. The engine being developed is a beta configuration Stirling engine with the power piston mounted on the cold heat exchanger's side. So, the rod that attaches the displacer, shown in pink, to the connecting rods goes through the power piston assembly shown in yellow. The guide rod that constrains the motion of the displacer is shown in grey. For a sense of scale, the flywheel shown has a diameter of 1550.8 mm.

III. Methodology

Previous displacers have been solid similar to [6], but the comparative displacer design for a 355.6 mm (14 in) diameter and 381 mm (15 in) tall displacer would put the weight of the displacer at 22 kg if the foam has a density of 288 kg/m^3 such as FR-4718 foam made by General Plastics shown in [7]. Most of this weight comes from metal fixtures to allow for assembly and the mounting of seals. This estimate, as well as future mass estimates, neglects the weight of fasteners on the displacer. Similar to the engine being developed, the displacer in [6] was designed with a 95°C hot source and a 2°C cold source.

The high weight of 22 kg would make the displacer have a large amount of inertia forces. Comparing the inertia forces with the pressure forces can be used to show their relative magnitude and importance. The engine under development has a 304.8 mm (12 in) diameter (D_{PP}) power piston that would transmit the pressure force of the engine to the mechanism. If the buffer pressure, or pressure acting on the crankcase side of the power piston, is correctly pressurized, then the power piston and its related mechanism will only feel the force caused by the pressure swing from mean engine pressure. This pressure swing depends on many factors such as the engine's compression ratio, working fluid temperatures during the cycle, the fill pressure of the engine, as well as the type of working fluid used. An accurate estimation of the pressure swing would need either a computer simulation software or complex numerical analysis that is outside the scope of this paper. The pressure swing (P_{swing}) from mean cycle pressure with a ten-atmosphere fill pressure was assumed to be 350 kPa. The inertia forces of the displacer will be based on a sinusoidal motion. Taking the mass of the displacer (m) and overestimating the running frequency (f) as three Hertz, the inertial forces of the displacer can be calculated as follows using the displacer's stroke (S). Differentiating the sinusoidal equation of motion twice the following equation for acceleration is derived:

$$a = 2\pi^2 S f^2 \sin(2\pi f t) \quad (1)$$

The equations for the maximum pressure force (F_P) and inertial force (F_I) on the mechanism are:

$$F_P = \frac{\pi}{4} D_{PP}^2 P_{swing} \quad (2)$$

$$F_I = ma = 2\pi^2 m S f^2 \quad (3)$$

For the given power piston geometry, the pressure force with a ten-atmosphere fill pressure, and its corresponding 350 kPa pressure swing, to be 25539 N using (2). For a ten-kilogram displacer and the overestimated running frequency and given stroke the inertial force is 452 N using (3), or 1.77% of the pressure force. While this may be negligible, the engine is also supposed to be able to run at one-atmosphere fill pressure so that multiple fill pressures can be tested when validating the new simulation software. If a linear relationship is assumed between fill pressure and pressure swing, a pressure swing of 35 kPa at a one-atmosphere fill pressure is determined. The corresponding pressure force being 2554 N. At this fill pressure the inertial force, at 452 N, makes up 17.67% of the force the mechanism must withstand. Considering the force the displacer must withstand at the upper range of fill pressure and non-negligible percent the inertia force contributes at the lower end fill pressures, a ten-kilogram target was determined to be a reasonable target for the new displacer. This leads to the investigation of a hollow displacer design.

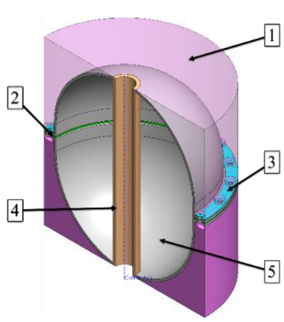

Figure 3: Section view of the domed displacer design, (1) foam dead volume reducer, (2) gasket, (3) plate for mounting bag seal, (4) main guide rod bar, (5) HM1189-0100 hemisphere

A common feature of the displacer designs being investigated is the ability to be disassembled. This limits the ability to use internal supports, so all designs being considered use fasteners to attach two halves of the displacer together as shown in Figure 3. The final design will also have a hole that is meant to bring the internal pressure of the displacer to the engine cycle mean pressure. This design results in the displacer having to resist the least amount of pressure, that being the pressure swings of the Stirling cycle at about 350 kPa with a ten-atmosphere fill pressure, instead of the absolute pressures within the engine which would be in the ballpark of 1350 kPa with a ten-atmosphere fill pressure. This is because attaching an apparatus to control the pressure within the displacer while the engine is running would be difficult. If instead the pressure were modified before installing the displacer, then every time the engine is repressurized to a new fill pressure for testing the displacer would have to be removed, repressurized, and reinstalled. So, a small hole is added so its internal pressure will naturally equalize to the engine's mean cycle pressure. Lastly, the displacer will need a guide rod through its center axis, mainly to alleviate pressure off the piston seals since the engine will be oriented horizontally due to space constraints.

With the assumption that the internal pressure of the displacer remains constant at the cycle mean pressure we can conduct a finite element analysis (FEA) simulation to determine if a flat metal face displacer is a viable solution. To reduce the dead volume, or volume within the engine not swept by either the displacer or power piston, caused by clearance volumes, the flat face of the displacer as well as the tube of the displacer would ideally deflect less than one millimeter under the pressure swings. The two materials chosen to be investigated is AISI 304 for its low thermal conductivity and strength and Aluminum 6061-T6 for its high strength to weight ratio.

The flat faces FEA was setup in SOLIDWORKS® by applying a pressure swing with a factor of safety of two on the top face, a cyclic symmetry condition around the central axis, a cylindrical fixed boundary condition on the inner diameter preventing radial translation, and a fixed boundary condition around the outer edge to simulate where the tube section will be welded to the displacer end caps. To allow for disassembly a 25.4 mm (one inch) flange will be added around the mid plane of the displacer. So, the internal pressure resisting body for a 355.6 mm (14 in) diameter displacer is 304.8 mm (12 in) in diameter. On the other hand, the FEA for the tube section will take the full 381 mm (15 in) height of the displacer since the flanges negligibly resist the deflection of the tube under pressure. The end caps simulation setup is shown in Figure 4 and the tube section's simulation setup is shown in Figure 5. The end caps were simulated using SOLIDWORKS® with a 1.27 mm (0.05 in) mesh size and the tube section were simulated with a 1.02 mm (0.04 in) mesh size.

With these conditions, an AISI 304 displacer would have a 7.94 mm (5/16 in) thick end cap and a 1.59 mm (1/16 in) thick tube section. On the other hand, an Aluminum 6061-T6 displacer would have a 9.53 mm (3/8 in) thick end cap and a 2.38 mm (3/32 in) thick tube section. Both displacers would need foam on the outside 25.4 mm (one inch) thick ring section to mitigate the dead volume the flange contributes. A rough mass estimate of these two displacers puts the AISI 304 displacer at 21.7 kg and the Aluminum 6061-T6 displacer at 10.1 kg. These two design's mass breakdowns are shown in Table I. Note the thickness for the tubes and their flanges are equal. The densities used were 8000 kg/m^3 for AISI 304 and 2700 kg/m^3 for Aluminum 6061-T6.

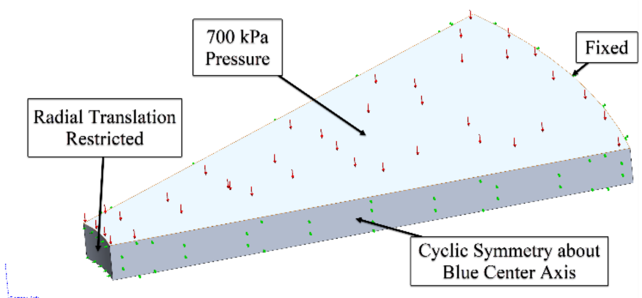

Figure 4: Flat face end caps simulation setup, with center axis in the bottom left.

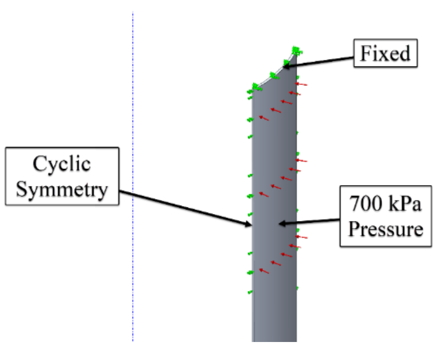

Figure 5: Tube section simulation setup. Only top half shown.

Table I: Mass breakdown for the flat face displacer designs

Component	AISI 304 Mass Contribution (kg)	Aluminum 6061-O Mass Contribution (kg)
Flat Face End Caps	9.27	3.75
Tubes with Flanges	6.89	2.55
Main Guide Rod Bar	2.70	0.91
Foam Ring	2.87	2.86
Total	21.7	10.1

These flat face designs can be improved by moving to a domed metal displacer design similar to the one shown in Figure 3. Using stock parts, commercial-(Toledo Metal Spinning) stock hemispheres are available with roughly a 304.8 mm (12 in) diameter as shown in [8]. Modelling a design in SOLIDWORKS® that uses their HM1189-0100 hemisphere gives an approximate mass of 9.92 kg if it is made of 4.7 mm (0.185 in) thick Aluminum 6061-O, a large amount of which comes from the added foam needed to remove the dead volume. The density for Aluminum 6061-O was 2700 kg/m^3. The foam used is the FR-4718 made by General Plastics as described in [7].

Based on manufacturing restrictions, 4.7 mm (0.185 in) thickness is the thinnest that could result from Toledo Metal Spinning's maximum blank size. Similar to the flat faces FEA a cyclic symmetry boundary condition was applied, with a fixed boundary condition where the flanges would meet, an external pressure force, and a radial fixed boundary condition for the guide rod's hole. These simulations used a 1.27 mm (0.05 in) mesh size and is shown in Figure 6. This resulted in the hemisphere being able to withstand the pressure swings at ten-atmosphere fill pressure with a factor of safety of 3.6, so some more weight could be saved if we lessened the factor of safety to two. An equivalent AISI 304 hemisphere with a thickness of 2.17 mm (0.0853 in) would withstand the same pressure swings with a factor of safety of 5.34. The weight breakdown of both designs are shown in Table II.

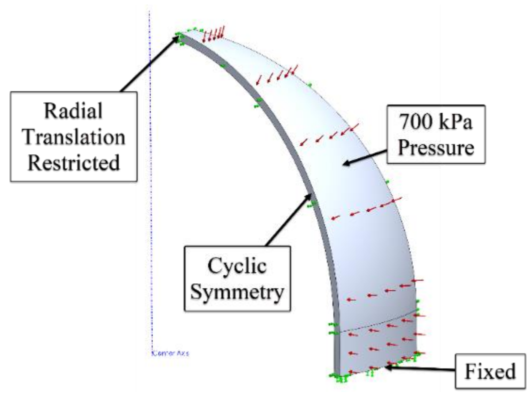

Figure 6: Simulation setup for hemisphere designs

Table II: Mass breakdown for the hemisphere designs

Component	AISI 304 Mass Contribution (kg)	Aluminum 6061-O Mass Contribution (kg)
Domes	3.33	4.89
Foam	4.50	4.50
Main Guide Rod Bar	2.55	0.86
Seal Mounting Plate	0.08	0.08
Total	10.46	10.44

Table III: Designs considered

	Flat Face Displacers				Domed Displacers				Solid	Sphere
Design Name	***Metal (1)***	***Composite Honeycomb (2)***	***Composite Sheet (3)***	***Internally Supported (4)***	***Outwards Hemisphere (5)***	***Inwards Hemisphere (6)***	***Pop Can Geometry (7)***	***Inverted Pop Can (8)***	***Full Foam (9)***	***Metal Sphere (10)***
Description	Flat metal endcaps with sheet metal tube similar to Figure 2	Flat honeycomb CFRP endcaps with a CFRP tube	Flat CFRP sheets for endcaps with a CFRP tube	Various internal support schemes with a flat metal endcap and sheet metal tube	Metal dome endcaps with convex side facing outwards like shown in Figure 3	Metal dome endcaps with convex side facing inwards	Imitate metal pop can geometry with sheet metal tube	Inverted metal pop can geometry with sheet metal tube	Full foam design similar to [6]	Two halves of a metal sphere
Considered in this paper	Yes	No	No	No	Yes	No	No	No	Yes	No

While many designs were considered as shown in Table III, most of which were not considered in depth due to either manufacturing concerns or constraints on the engine being developed. Design four was not considered because of the need for the ability to disassemble the displacer. Design nine was too heavy. Design six was not considered as it was similar to design five, but with more foam needed to reduce the dead volume while also requiring welds to attach the endcap to the tube. Design eight was not considered since it would be like design seven, but likely with more foam similar to the comparison between designs six and five. Design seven, while promising to have less foam than design five while likely resisting similar pressures, was not considered due to the manufacturing complexity and inability to find a stock part of the correct diameter and geometry. Design ten, while being similar to design five, was not considered since its lack of a cylindrical neck section meant a stock spherical design would restrict the height of the engine in the event the heat exchangers needed to be taller. Design three was then not considered by comparison with design two. Design three would likely not be much lighter than design two but would resist much lower pressures.

Of the designs not shown in this paper, only design two is being seriously considered. The low temperatures of the engine being developed mean that composite designs can be used within the engine. However, due to the variability of the material properties of composites, even amongst pre-impregnated composites, the verification of the design would require a costly test rig to make sure the displacer can handle the pressures once assembled. Because of this, design two will only be fully considered should the budget of the engine allow.

IV. Results

From Table IV we can see that the ten-kilogram target was not achieved for most designs, but a lot of weight can be saved with the dome designs if the foam can be mounted on the displacer cylinder instead of the displacer itself. However, a current limitation of the program being validated is that it relies on cylindrical elements, and so the displacer must have a flat face. Future development could allow for the incorporation of domed displacers, but until then the no foam hemisphere designs are not plausible for this engine's purpose.

Using (2) and (3) we can calculate each new design's inertia forces as a percent of the pressure force for the one-atmosphere fill pressure case. While none of the designs can make the inertia forces negligible at one-atmosphere fill pressure while still withstanding the pressure swings at a ten-atmosphere fill pressure, a lot of weight can be saved with most hollow displacer designs compared with the equivalent solid displacer at 22 kg.

It can also be seen in Table IV that the advantages of a hemisphere design are only utilized by materials with a lower strength to weight ratio. Moving to the hemisphere design made the aluminum design heavier due to the added foam to remove the dead volume created.

Table IV: Mass Comparison of the designs with corresponding factor of safety for ten-atmosphere fill pressure case and Inertia Force (F_I) expressed as percent of Pressure Force (F_P) at a one-atmosphere fill pressure

Design	**Weight (kg)**	**Factor of Safety**	**F_I (% of F_P)**
Aluminum 6061-T6 Flat Face	10.1	2	17.8
AISI 304 Flat Face	21.7	2	38.3
Aluminum 6061-O Hemisphere	10.44	3.6	18.4
AISI 304 Hemisphere	10.46	5.34	18.5
Aluminum 6061-O Hemisphere No Foam	5.94	3.6	10.5
AISI 304 Hemisphere No Foam	5.96	5.34	10.5

More weight could be saved with a more accurate estimation of the Guide Rod's bar. Especially for the AISI 304 designs, the guide rod's bar makes up a large chunk of the overall weight as seen in Table I and Table IV. Once the diameter of the guide rod is determined and corresponding linear bearings that will be fitted to the displacer, then this estimate can be improved. Additional weight can be saved with the hemisphere designs using a design study to lessen the factor of safety to two.

In conclusion, it is recommended to use the AISI 304 Hemisphere design. The high factor of safety means that a good chunk of weight can be saved if the factor of safety is reduced from 5.34 to two. In addition, AISI 304 has a lower thermal conductivity of 16 W/m*.K compared to Aluminum 6061-T6's 167 W/m*.K according to [9]. This limits the conduction losses between the expansion and compression spaces due to the displacer.

Further work to be done is a thermal resistance analysis. Even when reducing the AISI 304 hemisphere design's factor of safety to two, it is likely it is still mass competitive with the Aluminum 6061-T6 flat face design. Then, the thermal resistance analysis will be needed to determine which is better. While Aluminum has a much higher thermal conductivity than AISI 304, the small area of the aluminum tubes may make it more thermally resistive than the foamed dome of the AISI 304 with a larger conduction area. Other further work to be done after final dimensions are determined is a weight optimization design study, a more accurate mass analysis, and a final cost analysis.

Additionally, a carbon fiber reinforced polymer (CFRP) displacer design could be investigated. The low temperature inherent in most LTDSE's mean that CFRP and other composites can be utilized. However, the lack of consistency in material properties, even amongst pre-impregnated fabrics would mean that a costly testing procedure would need to be done to make sure the final design can handle the pressures. However, the mass saved with a CFRP displacer may be an advantage to mitigating the inertial forces the mechanism will have to endure.

V. Conclusions

This paper compared six hollow displacer designs using finite element analysis and numerical calculations and determined that an AISI 304 hemisphere design is the best design purely on a mass analysis. At a weight of 10.46 kg with a factor of safety of 5.34 to resist a pressure swing of 350 kPa, the hemisphere AISI 304 design could likely be made to be the lightest design of the flat face designs. Removing the foam that acts as a dead volume reduced would further reduce the weight, but the current limitations of the software being validated means that a flat face displacer would be needed.

Further work will be a design study optimizing the weight of the hemisphere designs, a more accurate mass and cost analysis once the designs have been optimized, a thermal resistance analysis to better understand the conduction losses through the displacer between the expansion and compression spaces and investigating the viability of a composite displacer design on a cost and manufacturing basis. The low temperatures inherent with LTDSEs mean that a composite displacer is a possible design solution that promises large weight savings but comes with the disadvantage of being expensive.

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