Proceedings of the Canadian Society for Mechanical Engineering International Congress 2021
CSME Congress 2021
June 27-30, 2021, Charlottetown, PE, Canada

A BAYESIAN APPROACH FOR IDENTIFYING NEW DYNAMICS IN MECHANICAL SYSTEMS

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Abstract—This paper is concerned with modeling methodologies using a combined physics-based and data-driven approach. The purpose of such models is to assist machinery fault diagnosis, root cause analysis, and system failure prediction. Particularly, we focus on capturing a class of faults, which, when developed in a physical system, would manifest themselves as "new dynamics". In other words, they are not present in healthy conditions. Examples include joints and connectors in multi-body systems. Under healthy conditions, they are typically represented by ideal boundary conditions. However, when faults have developed, they become entities with dynamic responses to input excitations. We propose a Bayesian inference-based methodology to detect the development of these new dynamics through model parameter uncertainty quantification. We demonstrate the effectiveness of this method through numerical experiments on a rotating mechanical system.

Keywords-physics-based modeling; Bayesian inference; fault diagnosis; parameter calibration, new dynamics

I. INTRODUCTION

Accurate and timely diagnosis of faults is an important goal of machine condition monitoring. Inability to diagnose a fault in machinery can result in increased maintenance costs, unplanned shutdowns, and even catastrophic component or system failure. To address this challenge, several types of diagnosis techniques have been developed based on advances in electromechanical modeling, system identification, signal processing, artificial intelligence, and data science [1-6].

Fault diagnosis methods can be grouped into signal-based and model-based methods. Signal-based methods are data-driven and attempt to diagnose faults by recognizing trends and patterns in the collected system data. Model-based methods use a physics-based or other type of model to generate predicted outputs that can be compared with measured data to assess the condition of the system. Physics-based models allow for prediction of the system's behaviour at operating conditions where limited or no measurement data exists. This is especially useful when studying fault or failure cases which are costly or impractical to run in practice, or for which limited data is available. In addition, incorporating faults into physics-based models allows for prediction of the progressing severity of the fault, as well as simulation of how the presence of the fault will affect the performance of other components in the system.

The way in which a fault is represented in a model provides insights into how they can be identified. Faults such as cracks and wear can be represented by changes in existing system parameters. Several methods exist to address the problem of calibrating or updating model parameters [7-10]. However, there are types of faults that involve changes in the configuration of the system, causing behaviours that cannot be captured by changing existing parameters in the original model. An example would be a joint that is assumed to be rigid in the original model, but then becomes loose and develops a new dynamic behaviour between the two parts of the joint. We will refer to these types of behaviours as "new dynamics". Development of these new dynamics in a real system is significant because these faults cannot be diagnosed by calibrating parameters in the original model. Diagnosis of faults associated with the development of new dynamics will be the focus of this work.

Fields such as adaptive control [7] and model-based fault diagnostics [8] address the problem of detecting changes in the system by monitoring or updating system parameters, while also accounting for other unmodeled dynamic effects. In adaptive control, these effects are referred to as unmodeled dynamics, and in model-based fault diagnostics, they are called model uncertainties. However, in each of these fields, these uncertainties or unmodeled effects are treated as disturbances, rather than quantities to be analyzed to gain information from. In the field of uncertainty quantification (UQ), these effects are referred to model discrepancy, and, following the pioneering work by Kennedy and O'Hagan in [9], are quantified in parallel along with parameter uncertainty. The most widely used approach in UQ is within a Bayesian framework [10-12] where the total error between the measured and simulated output is partitioned into system parameter uncertainty, model discrepancy due to missing dynamic effects, and random measurement noise.

Bayesian methods allows for the incorporation of existing knowledge about the system parameters by selecting appropriate prior distributions. They also provide a full probability distribution with associated statistical properties as the result of the methodology. This offers a more complete understanding of the uncertainty in the parameter compared to a single maximum likelihood value. In addition, unmodeled dynamic effects are collected into a model discrepancy function which can be analyzed to recover potential new dynamics in the system. However, a major ongoing challenge in this field is the identifiability problem, which makes it difficult in many cases to determine how much of the error between the measured and simulated outputs is due to parameter change, and how much is due to model discrepancy caused by effects such as new dynamics [10]. Since there may be multiple solutions that satisfy the formulation, additional information about the system is needed to specify a unique solution [13,14].

In this work, we will focus on the problem of identifying the presence of new dynamics faults in a system, based on applying the Bayesian approach first presented in [9]. We propose use of a physics-based model to identify new dynamics in a system by predicting the expected probability distribution of chosen calibration parameters. Characteristics of these distributions will be used to differentiate between cases where only the system parameters have changed, and cases where there is additional discrepancy due to new dynamics. We first present an example of a mechanical system, along with the model of a fault which involves new dynamics. The existing Bayesian inference formulation will then be introduced, followed by our proposed method for identifying the development of new dynamics, and preliminary results from the example system and fault. Finally, we will discuss the implications of these findings as well as future directions of this work.

II. Mechanical System & Fault Modeling

A rotating machinery system with coupled torsional-lateral vibrations will be used to demonstrate the proposed approach. We will present the equations of motion for the system under normal operating conditions, as well as an example of a fault that introduces new dynamics in the system.

A. Mechanical System and Equations

The mechanical system consists of a motor driving a load, joined by a coupling, as shown in Fig. 1. The motor, load and coupling are modeled as masses and rotary inertias, connected by springs with torsional and lateral stiffness. Bearings are located on either side of the motor and load components and are modeled as horizontal and vertical linear springs. Fig.1a shows the torsional system's components and degrees of freedom, and Fig.1b shows the lateral system.

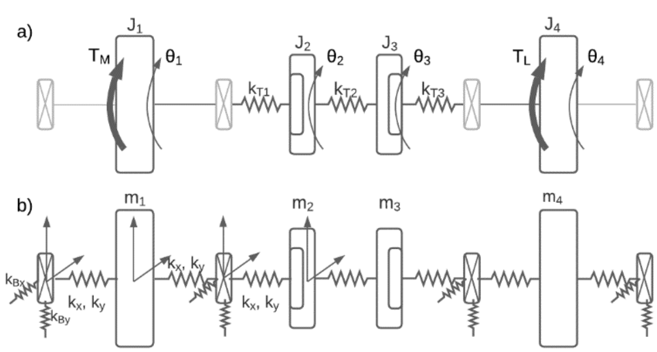

Figure 1. Schematic of rotating machinery system.

The torsional system is modeled as a 4-DOF lumped-parameter system comprised of rotary inertias, J_i, and torsional springs, k_{Ti}, with external driving torque T_M provided by the motor and a torque demand T_L at the load. The angular displacements at each inertia are given by θ_i. No damping is considered in the system. The equations of motion for the torsional system alone are:

$$J_1\ddot{\theta}_1 = -k_{T1}(\theta_1 - \theta_2) + T_m \quad (1a)$$

$$J_2\ddot{\theta}_2 = -k_{T1}(\theta_2 - \theta_1) - k_{T2}(\theta_2 - \theta_3) \quad (1b)$$

$$J_3\ddot{\theta}_3 = -k_{T2}(\theta_3 - \theta_2) - k_{T3}(\theta_3 - \theta_4) \quad (1c)$$

$$J_4\ddot{\theta}_4 = -k_{T3}(\theta_4 - \theta_3) - T_l \quad (1d)$$

The lateral system is modeled as horizontal and vertical DOFs at the four drivetrain component masses and at the four bearings, for a total of 16 lateral displacement DOFs. Mass imbalances in the motor and load shaft excite vibrations in the lateral system, which in turn excite the torsional system, resulting in coupled torsional-lateral vibrations [17]. The equations of motion for the torsional-lateral system can be derived using Lagrange's equations. In the interest of space, these equations will not be repeated here, as the focus of this work is on the development of new dynamics in the torsional system.

B. Modeling of Fault with New Dynamics

We will consider the development of new dynamics in the form of a stick-slip fault that occurs between the motor shaft and motor-side coupling hub. The sticking case, where the motor shaft and coupling hub move together rigidly, is given by Eq. (1b), and represents fault-free operation of the coupling connection. In the slipping case, the component J_2 would behave as separate inertias, denoted J_{2a} and J_{2b}, which move relative to each other with angular displacements θ_{2a} and θ_{2b}, and a friction torque T_f acting between them:

$$J_{2a}\ddot{\theta}_{2a} = -k_{T1}(\theta_{2a} - \theta_1) - T_f \quad (2a)$$

$$J_{2b}\ddot{\theta}_{2b} = -k_{T2}(\theta_{2b} - \theta_3) + T_f \quad (2b)$$

Whether the connection is in a sticking or slipping condition can be predicted by comparing the interface torque, T_f, between the shaft and coupling hub to a maximum friction torque value. The friction torque between the two inertias can be calculated by equating the angular accelerations in Eq. (2a) and (2b), assuming the connection begins in a "sticking" condition.

$$\frac{1}{J_{2a}}\left[-k_{T1}(\theta_{2a} - \theta_1) - T_f\right] = \frac{1}{J_{2b}}\left[-k_{T2}(\theta_{2b} - \theta_3) + T_f\right] \quad (3a)$$

$$T_f = \left[-\frac{k_{T1}}{J_{2a}}(\theta_{2a} - \theta_1) + \frac{k_{T2}}{J_{2b}}(\theta_{2b} - \theta_3)\right]\left(\frac{J_{2a}J_{2b}}{J_{2a}+J_{2b}}\right) \quad (3b)$$

This friction torque can be used in a logical condition to determine whether there is sticking or slipping in the coupling [16]. If the friction torque exceeds the maximum friction torque T_{max}, that the connection can tolerate, the two components will slip relative to each other. If not, they will stay connected and move together. The stick-slip phenomenon occurs when the system quickly alternates between the sticking and slipping conditions. This may occur under a variable torque demand from the load, which is a typical situation for many types of rotating machinery.

During slipping, the torsional system behaves as a 5-DOF system, with additional dynamics that are not present in the original system model. It is clear that these dynamics cannot be accounted for by calibrating parameters in the original system.

In the next sections, we will present the fundamentals of the existing Bayesian inference approach for parameter calibration and present our proposed methodology for applying it to recognizing the emergence of new dynamics.

III. Bayesian Inference for Parameter Calibration

The Bayesian approach to parameter calibration and uncertainty quantification is based on Bayes' Theorem [15]. We briefly note that in most literature in the area of Bayesian inference, the set of calibration parameters is denoted as θ. However, as we have used θ and its derivates in earlier sections to represent the rotational degrees of freedom of the mechanical system, we will use the symbol φ instead for the calibration parameter set.

In the context of parameter calibration, the Bayesian approach is used to calculate the probability distribution of the value one or more parameters φ, given some known, measured data, y. This distribution, known as the posterior p(φ|y), is, by Bayes' Theorem, proportional to the product of a chosen prior distribution, p(φ), and a calculated likelihood function, p(y|φ):

$$p(\varphi|y) \propto p(\varphi)p(y|\varphi) \tag{4}$$

The prior distribution reflects our existing knowledge about the probability distribution of a parameter, and the likelihood represents how likely it is that the collected data is true, given a certain value of the parameter φ. The likelihood is based on the difference between the measured output, denoted as y^e, and the simulated output at a given parameter value, denoted as $y^m(\varphi)$. The smaller this difference, the higher the likelihood value. The Gaussian likelihood function is a common choice. The posterior distribution represents how likely each value of φ is to be true, given the collected data y. It is often analytically intractable, and is instead calculated numerically through sampling methods such as Markov-chain Monte Carlo (MCMC) methods [15].

An extended version of this Bayesian approach for parameter calibration was first introduced in [9], and expanded in works such as [10,11]. The extended formulation includes a term for model discrepancy or model bias, which accounts for errors and uncertainty due to modeling simplifications, nonlinearities, and other unmodeled effects. It is expressed as:

$$y^e(x) = y^m(x,\varphi) + \delta(x) + \varepsilon, \tag{5}$$

where $y^e(x)$ is the measured output data, $y^m(x,\varphi)$ is the simulated output from the model, δ(x) is the model discrepancy function, and ε is the random measurement noise. Here, x represents the system's control inputs or operating parameters. The procedure for calculating the posterior distribution of φ is computationally complex, so in the interest of space, we will not present the details here. We refer the reader to works in [9-11] for a more thorough discussion of existing methods.

A major limitation of the current Bayesian approach is the lack of identifiability [10,13] between the effects of changes in the parameters, or the development of new dynamics, which would be captured by the model discrepancy function δ(x). In the context of fault diagnosis, discerning whether a fault in the system is due to a parameter change or the development of new dynamics is often difficult.

IV. Methodology & Results

In this section, we will present our proposed methodology and apply it to the example mechanical system and fault introduced in section II. The goal is to recognize a change in the system output as being due to a fault that involves new dynamics rather than a change in an existing parameter.

A. *Proposed Methodology*

Our proposed method involves applying the Bayesian inference approach to obtain the posterior distribution of a parameter in a situation where new dynamics is known to exist. The central idea of this method is that parameter calibration alone will not be able to accurately account for the new dynamic behaviour, and should result in an inconsistent posterior distribution. Use of a physics-based model will allow us to simulate the behaviour of this fault in the system, as well as the expected posterior distribution for certain calibration parameters.

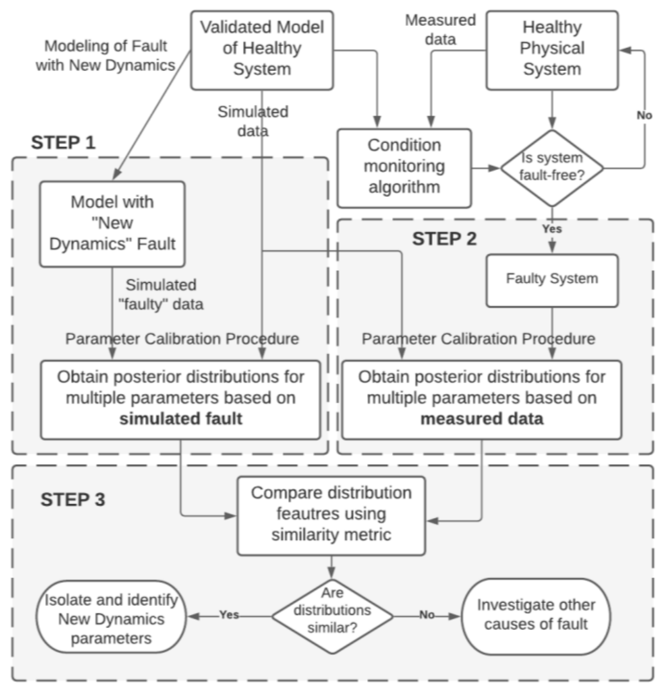

Figure 2. Flowchart of proposed methodology.

Fig. 2 shows a flowchart outlining the steps for our proposed methodology. In Step 1, we choose a type of new dynamics fault that we wish to be able to diagnose, should it occur in the real system. Using the original model of the healthy system, and the model with the new dynamics fault, we perform parameter calibration to obtain predicted posterior distributions. In other words, we simulate the fault, and compute the expected results if we were to calibrate different parameters to account for the change in the system due to this fault.

In Step 2, once we determine that the real system contains a fault, we perform a similar set of parameter calibrations as in Step 1. We obtain the posterior distributions from parameter calibration based on the real data from the system with an unknown fault. In Step 3, we compare the posterior distributions from Steps 1 and 2 using some similarity metric, and decide whether or not the unknown fault in the system is likely to be due to the chosen new dynamics fault.

B. Results

In this section, we will present and compare the results for two representative cases in order to demonstrate our proposed methodology.

In the first case, we will modify the value of a parameter in the original system, and use the Bayesian inference approach to calculate the posterior distribution of the parameter. This case will be used as a baseline for what the posterior should look like if calibrating the chosen parameter can accurately account for the change in the system. In the second case, we will simulate the system's behaviour with the addition of new dynamics from the stick-slip fault. We will then perform the same Bayesian analysis to try and account for the change in the system behaviour by calibrating different parameters. As mentioned earlier, we expect that this second case will yield inconsistent results.

In the base case, we change the parameter representing the torsional stiffness of the motor shaft (k_{T1} in Fig. 1) from 3E6 N/m to 3.25E6 N/m in the system, and then attempt to calibrate the value of the parameter using the existing Bayesian approach discussed earlier. The prior distribution is assumed to be a normal distribution with a mean equal to the original parameter value and a standard deviation of 0.5E6 N/m. The likelihood function is calculated based on the difference between the simulated system output with the new torsional stiffness, and the model output at a range of φ values. In each case, the horizonal vibration in the non-drive end motor bearing was used as the output.

In Fig. 3, we can see that the likelihood function is highest around the new value of k_{T1} (3.25E6 N/m), with a relatively narrow shape, indicating high confidence in the value of the parameter. The posterior distribution, obtained by a Monte Carlo sampling method, is also highest near the new true value of k_{T1}. This base result shows that if the correct parameter is chosen to account for a change in the system's behaviour, the results of the likelihood function and posterior distribution should yield consistent and accurate results. Note that for these figures, the priors and posteriors are probability density functions, while the likelihood is not, and has been scaled to fit on the same axis.

In the second case, we will simulate the original system with the addition of a stick-slip fault, and perform the same parameter calibration on this new system. We then show the results for this case, where parameter calibration alone should not be sufficient to account for the discrepancy between the system and model output.

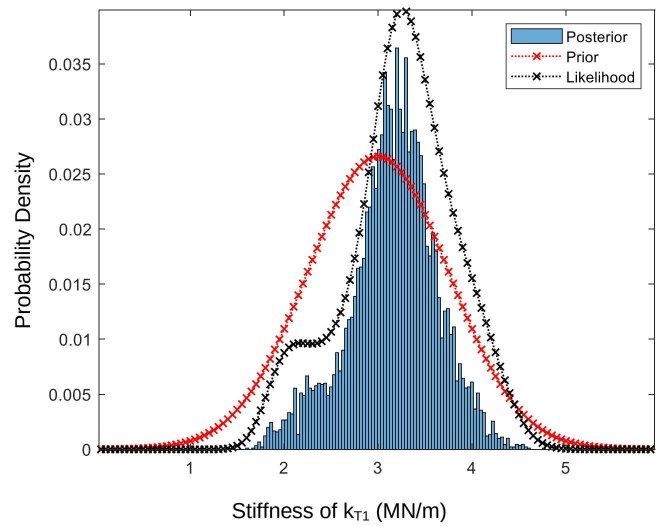

Figure 3. Calibration results for parameter change in system.

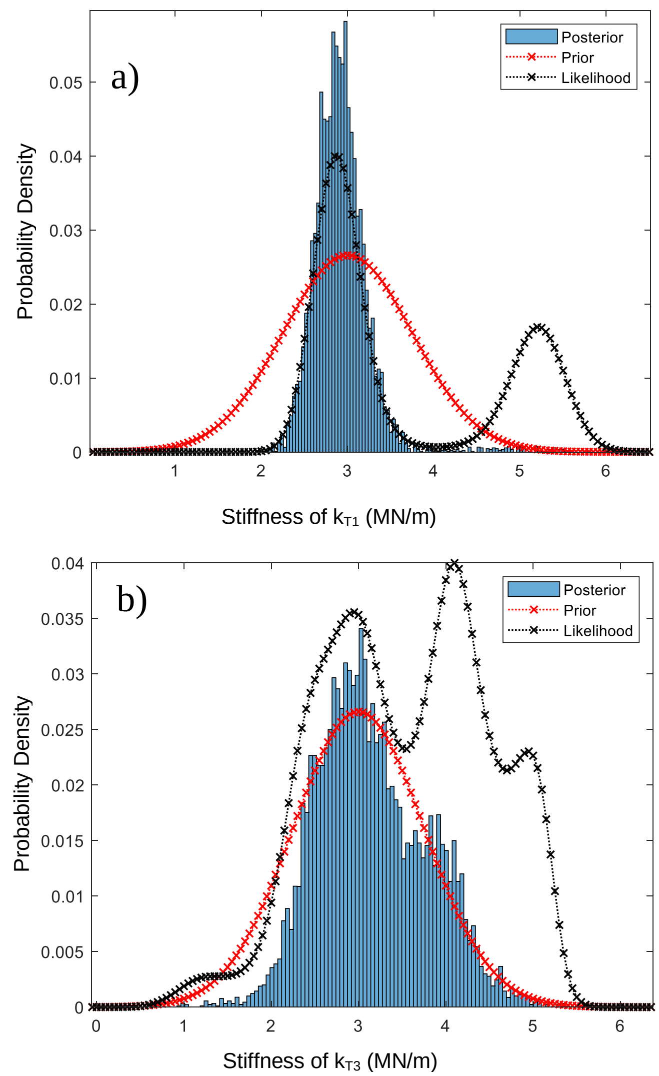

Figure 4. Calibration results for (a) k_{T1} (b) k_{T3} with new dynamics fault in system.

For the results in Fig. 4a) and b), a stick-slip fault was induced in the system, and the calibration procedure was performed with the motor shaft stiffness k_{T1}, and load shaft stiffness k_{T3} (refer to Fig. 1) as the calibration parameters. In each case, the likelihood function has multiple distinct peaks, some of which do not correspond to either the original or new parameter values. The posterior distribution shows similar characteristics as the likelihood function, but has very low values where the chosen prior distribution is also low. Since the posterior distribution is sensitive to the exact choice of the prior, the likelihood provides more complete information about whether or not the parameter calibration can accurately account for the changes in the system's behaviour.

This case shows that the results obtained by using parameter calibration to account for new dynamics behaviour are not consistent with the actual state of the system. Similar posterior distributions can be calculated for other parameters in the original model.

Inspection of the likelihood and posterior for inconsistencies constitutes a qualitative approach to infer the presence of new dynamics. For a more thorough, quantitative analysis, these simulated results can be compared to the results obtained from applying the Bayesian approach to experimental data in which new dynamics are thought to exist. This is shown in Steps 2 and 3 of the methodology flowchart in Fig. 2. A potential method for doing this is using a measure such as the Hellinger distance or similar metric to compare the similarity of two probability distributions. If the same new dynamics fault as was simulated has developed in the real system, the likelihood function and posterior distribution should present similar characteristics.

V. Conclusions & Future Work

The results presented in this work have demonstrated an approach which allows for identification of new dynamics in a system. We applied parameter calibration to a system where an existing system parameter had changed, as well as one where a fault with new dynamics was simulated, and calculated the predicted posterior distributions for both cases. The features of the posterior distributions for each case were compared. The results showed that attempting to account for the development of new dynamics by performing parameter calibration yielded inconsistent results in the likelihood function and posterior distribution. Having a physics-based model allowed us to simulate and predict the effect of the new dynamics and associated fault. In addition, we were able to predict what the posterior distribution would look like if calibration of the system was performed using a given parameter.

The next steps in this work would be to extend the methodology to cover a range of different systems and classes of new dynamics and faults. A further extension of the work would involve updating the original model by integrating a submodel of the new dynamics. This would require isolation of the fault submodel, and identification of the key parameters of the new dynamics model.

Acknowledgments

The authors wish to acknowledge the financial support for this research provided by the Natural Sciences and Engineering Research Council (NSERC) through the PGS-D scholarship program.

References

[1] Q. Sun, P. Chen, D. Zhang, F., Xi, Pattern recognition for automatic machinery fault diagnosis, Journal of Vibration and Acoustics, Vol. 126, pp. 307-316, April 2004.

[2] Y. Peng, M. Dong, M.J. Zuo, Current status of machine prognostics in condition-based maintenance: a review, International Journal of Advanced Manufacturing Technology, 50, pp. 297-313, 2010.

[3] S. Zhang, B. Wang, Deep learning algorithms for bearing fault diagnostics – a comprehensive review, IEEE Access, February 2020.

[4] N.I. Azeez, A.C. Alex, Detection of rolling element bearing defects by vibration signature analysis: a review, International Conference on Magnetics, Machines & Drives, 2014.

[5] M. Cerrada, R. Sanchez, C. Li, et.al, A review of data-driven fault severity assessment in rolling bearings, Mechanical Systems and Signal Processing, 99, pp. 169-196, 2018.

[6] Q. Sun, Y. Tang, Y. Ji, Feature extraction with discrete wavelet transform for drill wear monitoring, Journal of Vibration and Control, 11 (11), pp. 1375-1396, 2005.

[7] H. Jin, Z. Liu, A dynamic parameter identification method for flexible joints based on adaptive control, IEEE/ASME Transactions on Mechatronics, Vol.23, No.6, December 2018.

[8] Z. Kulesza, T. Sawicki, A.L. Gyekenyesi, Robust fault detection filter using linear matrix inequalities' approach for shaft crack diagnosis, Journal of Vibration and Control, 19(9), pp. 1421-1440, 2012.

[9] M.C. Kennedy, A. O'Hagan, Bayesian calibration of computer models, Journal of the Royal Statistical Society, 63, Part 3, pp. 425-464, 2001.

[10] P.D. Arendt, D.W. Apley, W. Chen, Quantification of model uncertainty: calibration, model discrepancy, and identifiability, ASME. Journal of Mechanical Design, 134(10): 100908, October 2012.

[11] X. Wu, T. Kozlowski, H. Meidani, K. Shirvan, Inverse uncertainty quantification using modular Bayesian approach based on Gaussian process, Part 1: Theory, Nuclear Engineering and Design, 335, pp. 339-355, 2018.

[12] D.E. Ricciardi, O.A. Chkrebtii, S.R. Niezgoda, Uncertainty quantification accounting for model discrepancy within a random effects Bayesian framework, Integrating Materials and Manufacturing Innovation, 9, pp. 181-198, 2020.

[13] P.D. Arendt, D.W. Apley, W. Chen, D. Lamb, D. Gorsich, Improving identifiability in model calibration using multiple responses, ASME. Journal of Mechanical Design, 134(10): 100909, October 2012.

[14] J. Brynjarsdottir, A. O'Hagan, Learning about physical parameters: the importance of model discrepancy, Inverse Probability, 30(11):114007, 2014.

[15] A. Gelman, J.B. Carlin, Bayesian Data Analysis, 3rd ed., Chapman and Hall, 2013.

[16] L. Li, Z. Lu, X.L. Liu, T. Sun, X. Jing, W.B. Shangguan, Modeling and analysis of friction clutch at a driveline for suppressing car starting judder, Journal of Sound and Vibration, 424, pp. 335-351, 2018.

[17] I. Perera, Theoretical and Experimental Study of Coupled Torsional-Lateral Vibrations in Rotor Dynamics, PhD Thesis, University of Calgary, December 1998.