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NUMERICAL STUDY OF MIXED CONVECTIVE HEAT TRANSFER FROM NARROW SIDE-UP TWO-SIDED ISOTHERMAL FLAT PLATES

Mohamed Elkhmri, Patrick H. Oosthuizen*
Department of Mechanical and Materials Engineering, Queen's University, Kingston, Canada
*patrick.oosthuizen@queensu.ca

Abstract—Many studies of mixed convective heat transfer from heated flat plates which are in the same plane as the forced flow over the plate surface are available, most of these studies dealing with wide plates. However, there exist practical situations in which there is mixed convection from thin two-sided plates that lie in a vertical plane in a horizontal forced flow, the plate thus being at right angles to the forced flow. In some such cases the plates are also narrow, and the relative plate width is expected to have a significant influence on the heat transfer rate in such a case. The present study therefore numerically investigates how the relative width of the plate affects the mixed convective heat transfer rate from a thin narrow side-up rectangular plate. It has been assumed that the plate surfaces are isothermal and at the same temperature. The Boussinesq approach has been adopted. Conditions under which laminar, transitional, and turbulent flows exist have been considered, the standard *k*-epsilon turbulence model being used with buoyancy force effects being accounted for. The solution has been obtained using ANSYS FLUENT©. The mean heat transfer rates from the surfaces of the plate have been expressed in terms of Nusselt numbers based on the length of the plate. These Nusselt numbers are dependent on the Rayleigh number and the Reynolds number based on the plate length, on the ratio of the plate width to the plate length, and on the Prandtl number. Results have only been obtained for a Prandtl number of 0.74. Variations of the Nusselt numbers with Rayleigh number and with Reynolds number for various dimensionless plate widths have been obtained. The results show that the effect of the plate width can be very significant.

Keywords- Mixed convection, Combined convection, Narrow plate side up, Horizontal flow, Numerical

I. INTRODUCTION

In some situations in which there is a forced flow over a body, and in which temperature differences exist in the flow, the buoyancy forces that arise due to density differences resulting from the temperature differences have a significant effect on the flow despite the presence of the forced flow. Such flows are termed mixed or combined natural and forced convective flows. There have been a number of studies of mixed convective heat transfer from vertical flat plates, e.g. [1-12], from horizontal plates, e.g. [13-15], and from inclined plates, e.g. [16-20]. Basically, all of these studies of mixed convective heat transfer have been concerned with wide plates, i.e., with situations in which the flow over the plate can be assumed to be two-dimensional. However, several practical situations exist that effectively involve heat transfer from narrow plates, i.e., from plates whose width is relatively small compared to their length. There have been a very limited number of studies that deal with mixed convective heat transfer from narrow plates, e.g. [21, 22].

In the present study attention has been given to mixed convective heat transfer in a situation that occurs in practice but which has not been considered in any of the papers mentioned above. The situation here considered involves a narrow thin rectangular two-sided plate which lies in a vertical plane and which is placed in a horizontal forced flow that is parallel to the plate surfaces. The situation considered is as shown in Fig. 1. In such a situation, the buoyancy forces in the flow that result from the temperature differences in the flow are in the vertical direction and therefore act at right angles to the forced flow. The heat transfer rate in this situation has been numerically studied in the present work. The present study is part of a series of inter-related studies of mixed convective heat transfer from narrow two-sided heated surfaces. Previous studies that are part of this broad investigation are described in [21, 22].

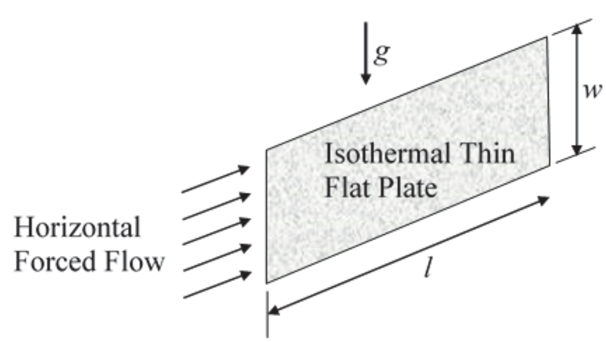

Figure 1. Flow situation considered in present study.

II. SOLUTION PROCEDURE

It has been assumed that the two-sided flat plate here being considered is very thin, i.e., that the plate effectively has no thickness. The two surfaces of the plate have been assumed to be isothermal and at the same temperature. The flow has been assumed to be steady and to be symmetrical about the vertical center-plane through the plate, i.e., to be the same on the two sides of the plate. The Boussinesq approach has been adopted in dealing with the buoyancy forces. Conditions under which laminar flow, transitional flow, and turbulent flow potentially exist have been considered, and to deal with this, the standard *k*-epsilon turbulence model with full account being taken of buoyancy force effects has been used. This turbulence model has been applied under all conditions considered. The boundary conditions applied on the outer surfaces of the solution domain were: (a) on the upstream surface that is normal to the forced flow it was assumed that the flow velocity was uniform and the flow was normal to the surface, (b) on all other outer surfaces it was assumed that the pressure was uniform and the flow was normal to the surface. The solution has been obtained using the commercial CFD solver ANSYS FLUENT©. Extensive grid- and convergence criterion independence testing was undertaken. This indicated that the mean heat transfer results presented here are to within 1% independent of the number of grid points and of the convergence-criterion used.

III. RESULTS

Attention has here been restricted to the mean heat transfer rate from the surfaces of the plate, the mean heat transfer rates from the two sides of the plate being the same. This mean heat transfer rate has been expressed in terms of a mean Nusselt number based on the plate length, l, i.e., if q' is the mean heat transfer rate per unit surface area (the surface heat flux) from each surface of the plate the following Nusselt number has been used:

$$Nu_{\mathrm{mean}} = q'\, l \,/\, k(T_w - T_f) \tag{1}$$

where T_w and T_f are the temperature of the isothermal plate surfaces and of the undisturbed fluid far from the plate, respectively. These Nusselt numbers will depend on the following:

- The Rayleigh number, *Ra*, based on the plate length, *l*,
- The Reynolds number, *Re*, based on the plate length, *l*, and on the forced velocity in the undisturbed flow ahead of the plate, *U*,
- The relative width of the plate, $W = w/l$,
- The Prandtl number, *Pr*.

Results have only here been obtained for $Pr = 0.74$, i.e., effectively the value for air at ambient conditions.

Typical variations of Nu_{mean} with *Ra* for various values of *Re* for *W* values of 0.1, 0.3, and 0.6 are shown in Figs. 2, 3. and 4. It will be seen that at the lower values of *Ra* the flow at the higher values of *Re* is essentially purely forced convection whereas at the higher values of *Ra* and lower values of *Re* the flow tends towards being purely natural convective, mixed convection existing between these two limits. These figures also show that the Nu_{mean} numbers tend to decrease as the value of *W* increases, this being particularly significant at the lower values of *Re* considered.

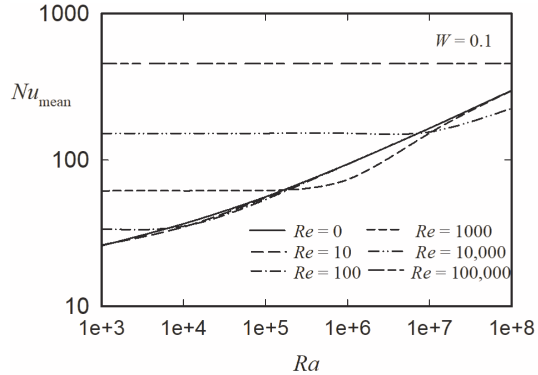

Figure 2. Variation of the mean Nusselt number, Nu_{mean}, with Rayleigh number, *Ra*, for various values of the Reynolds number, *Re*, for a dimensionless plate width, *W*, of 0.1

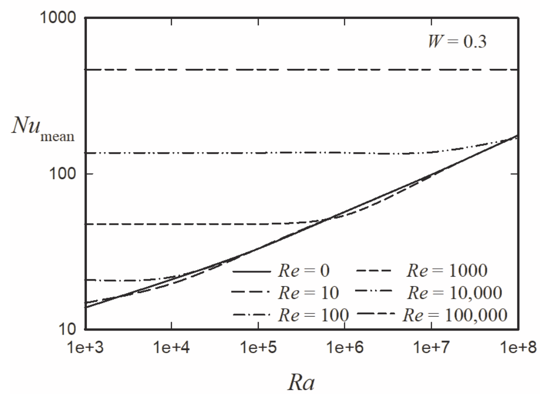

Figure 3. Variation of the mean Nusselt number, Nu_{mean}, with Rayleigh number, *Ra*, for various values of the Reynolds number, *Re*, for a dimensionless plate width, *W*, of 0.3.

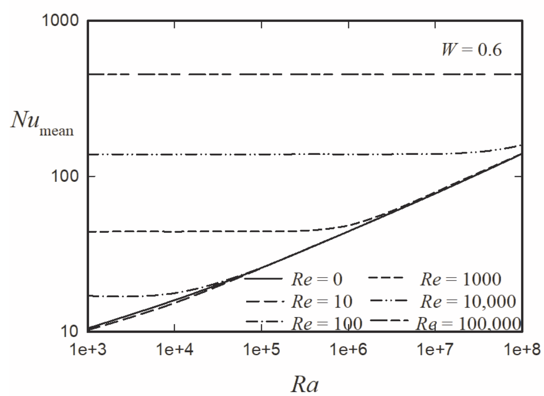

Figure 4. Variation of the mean Nusselt number, Nu_{mean}, with Rayleigh number, *Ra*, for various values of the Reynolds number, *Re*, for a dimensionless plate width, *W*, of 0.6.

Typical variations of Nu_{mean} with Re for various values of Ra for W values of 0.1, 0.3, and 0.6 are shown in Figs. 5, 6 and 7. It will be seen that at the lower values of Re the Nusselt number is effectively independent of Re, i.e., purely natural convective flow exists. At the higher values of Re the Nusselt numbers for all values of Ra considered approximately fall on a single curve, i.e., effectively purely forced convection exists. The results also show the somewhat complex Nusselt number variation that exists in the mixed convection region. These figures again also show that the Nu_{mean} numbers tend to decrease as the value of W increases, this being particularly true in the purely natural convective flow region.

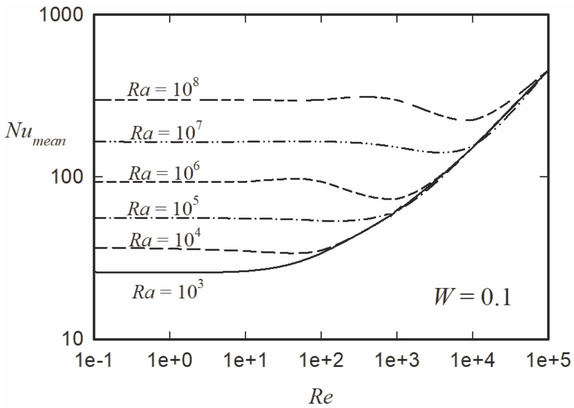

Figure 5. Variation of the mean Nusselt number, Nu_{mean}, with Reynolds number, Re, for various values of the Rayleigh number, Ra, for a dimensionless plate width, W, of 0.1.

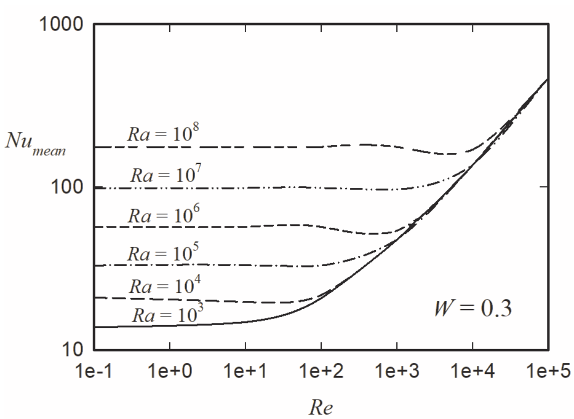

Figure 6. Variation of the mean Nusselt number, Nu_{mean}, with Reynolds number, Re, for various values of the Rayleigh number, Ra, for a dimensionless plate width, W, of 0.3.

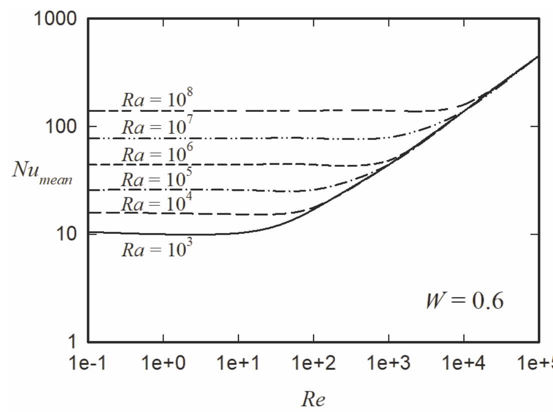

Figure 7. Variation of the mean Nusselt number, Nu_{mean}, with Reynolds number, Re, for various values of the Rayleigh number, Ra, for a dimensionless plate width, W, of 0.6.

The effect of W on the Nusselt number values is further illustrated by the typical variations of Nu_{mean} with W for various values of Ra for Re =1000 and for Re = 10 shown in Figs. 8 and 9 respectively. These figures again illustrate how the Nusselt number values increase as W decreases and show that this effect increases in magnitude as the Rayleigh number increases.

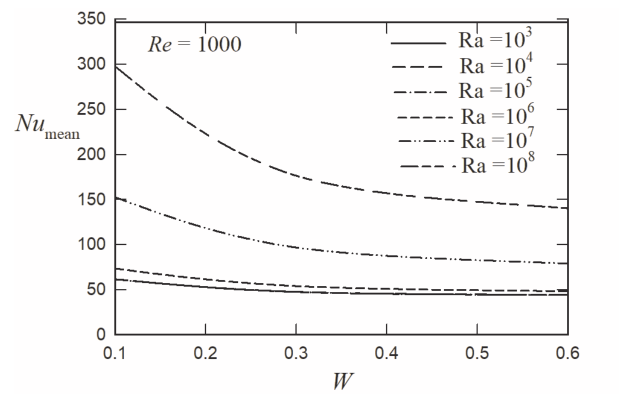

Figure 8. Variation of the mean Nusselt number, Nu_{mean}, with dimensionless plate width, W, for various values of the Rayleigh number, Ra, for a with Reynolds number, Re, of 1000.

IV. Conclusions

The results of the present study clearly show that with mixed convective heat transfer from narrow edgewise vertical plates, the plate width does have a significant influence on the heat transfer rate from the plate, the heat transfer rate from the plate increasing as the relative width of the plate, W, decreases. This effect increases as the purely natural convective flow region is approached. For the conditions under which mixed convective flow exists the effect of the relative width of the plate on the heat transfer rate was found to be significant for values of W less than approximately 0.6. In the purely forced convection region where there is essentially no flow induced normal to the direction of the upstream forced flow, the plate width has an almost negligible effect on the heat transfer rate.

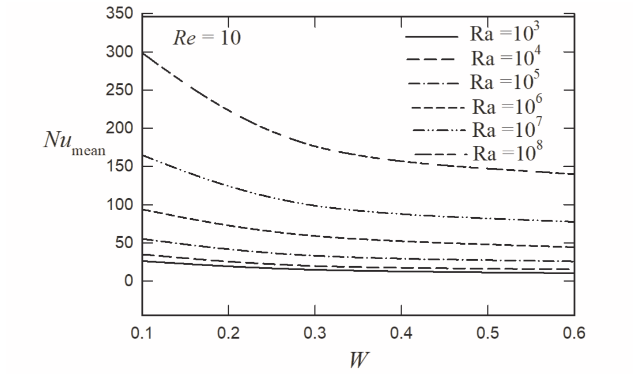

Figure 9. Variation of the mean Nusselt number, Nu_{mean}, with dimensionless plate width, W, for various values of the Rayleigh number, Ra, for a with Reynolds number, Re, of 10.

Acknowledgment

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