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An Adaptive Variable Structured-Based Filter using Multiple Models

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Abstract—The Kalman filter (KF) is the most well-known estimation strategy which yields the optimal solution in terms of error to the linear quadratic estimation problem for linear, known systems in the presence of Gaussian noise. While the KF is effective under the stated conditions, it lacks robustness to disturbances which are prevalent in real-world applications. Since its inception 60 years ago, there have been numerous variants of the KF developed to accommodate nonlinear systems, non-Gaussian noise, and modeling uncertainties. The smooth variable structure filter (SVSF) is as an alternative to the KF with improved robustness, especially in the case of external disturbances. It is based on sliding mode techniques that offer robustness at the cost of optimality. The static multiple models estimator incorporates several possible operating modes and generates an estimation that is weighted based on the likelihood of each mode. This paper introduces an adaptive formulation of the SVSF based on static multiple models, and applies the developed strategy on an electrohydrostatic actuator.

Keywords-State and parameter estimation; Kalman filter; smooth variable structure filter; robustness; static multiple models

I. INTRODUCTION

State estimation involves the extraction of important values known as states from noisy measurements [1]. States change over time and are typically governed by equations that describe system dynamics [2]. Estimators are a type of filter that also include smoothers and predictors. The purpose of estimation is to minimize the error (difference between the actual and estimated state values) while simultaneously reducing the effects of noise and being robust to disturbances [2]. Disturbances and noise are typically present in measurements, and may be caused by the sensor quality as well as environmental factors. System uncertainties may be caused by an inaccurate model and/or variations and nonlinearities in the physical system parameters. Reliable estimates of state parameters are necessary for safely and accurately controlling electro-mechanical systems in real-time. When system dynamics are changed abruptly in the presence of faults, adaptive estimation strategies can be used to mitigate inaccurate estimation.

Kalman expanded on the research of his predecessors and introduced a new solution to linear filtering and tracking problems [3]. He derived a filter that utilized linear models and measurements to yield a mathematically optimal estimate based on strict assumptions. This filter later became known as the Kalman filter (KF). Since the KF is not robust to disturbances or modeling uncertainties, variations of the KF have been formulated to account for them [3].

Another branch of state and parameter estimation methods that developed in parallel to the KF and its variants is known as sliding mode observers (SMOs), which are based on variable structure techniques [4, 5, 6]. Variable structure techniques consider systems that contain discontinuities in the system that describe their dynamic states. Discontinuity hyperplanes are used to divide the state space into different regions; within these regions, the equations used to describe the system are continuous [7, 8]. The name 'variable structure' is chosen since system dynamics may be mathematically described by a finite number of equations.

Variable structure theory provided the foundation for variable structure control. In VSC, the control input is formulated as a discontinuous state function, such that discontinuity hyperplanes are introduced [7, 8]. The most well-known type of VSC is known as the sliding mode controller (SMC) [5, 9]. SMC makes use of a discontinuous switching plane along a desired state trajectory, which is called a sliding surface. The primary objective for the SMC is to minimize state errors by sliding the states along the surface. A switching gain is used to push the states towards the defined sliding surface. Once the state values are on the surface, known as a sliding mode, the state slides along the defined surface [9]. Although the switching effects bring robustness and stability to the control process, it also introduces high-frequency switching known as chattering [10]. Quite often a boundary layer is introduced in an effort to smooth out and saturate the control signal [9]. Prior to the 1980s, VSC and SMC methods were only considered in the continuous-time domain [11]. In 1985, a discrete-time formulation of SMC was presented [12]. A stability condition was provided shortly afterwards and is now typically used in the design of discrete controllers [13, 14].

In the 1980s, SMOs were developed based on sliding mode and variable structure theory [11, 15]. Sliding mode observers minimize the error with the help of a switching function similar to VSC and SMC [16]. Observer gains are calculated based on the error (matching the system and observer outputs), and moving the error surface to zero [16]. Most SMOs apply a discontinuous signal to the estimates in order to keep them bounded to an area of the surface [11]. The motion consists of three phases: reachability, injection, and sliding [11, 17]. The reachability phase consists of forcing the estimates to the sliding surface from some initial conditions, in a finite period of time [11]. Once within a defined area of the surface (called an existence subspace), both the injection and sliding phases are present. The sliding phase forces the estimated errors to slide along a hyperplane towards the origin [11]. The injection phase consists of preventing the estimate from leaving the existence subspace; keeping it bounded within an area of the sliding surface [11]. According to [11, 15, 18], the action of the injection phase enables the observer to be robust enough to overcome uncertainties, modeling errors, and nonlinearities present in the system. A number of SMOs have been developed based on these principles. The most notable observers include those introduced by Slotine et al. [8, 19], Walcott et al. [20, 21], Edwards et al. [18], and later by both Tan and Edwards [22]. SMOs have been applied to estimation problems, and fault detection and isolation [11].

A new type of filter called the smooth variable structure filter (SVSF) was presented in 2007 based on sliding mode and variable structure techniques [2, 11, 23]. The SVSF is formulated as a predictor-corrector estimator similar to the KF, but utilizes a gain structure based on sliding mode techniques. The gain calculated by the SVSF is based on the measurement errors (known as innovation) and a switching term [23]. Similar to SMOs, the switching gain structure improves stability and robustness of the estimation process by bounding the state estimates close to the true trajectory [24, 25]. The SVSF presented in [23] did not contain a state error covariance derivation which is an important feature for estimation strategies (it is another performance indicator). A state error covariance function was introduced and expanded in [24, 26, 27] which vastly improved the number of useful applications for the SVSF [28, 29, 30]. Other improvements to the SVSF include: fault detection using chattering, higher-order implementations, and tracking multiple targets [11, 31, 32, 33, 34]. The SVSF has demonstrated robust performance on a number of different estimation problems [3]. Most recently, a new type of filter called the sliding innovation filter (SIF) was introduced in [35]. The SIF is based on similar concepts to the SVSF, but offers a simpler formulation with improved results.

In this paper, a new adaptive formulation of the SVSF is considered. The static multiple models estimator (SMM) incorporates several possible operating modes and generates an estimate that is weighted based on the likelihood of each mode. This strategy is attractive for fault detection and diagnosis problems in mechatronics and other engineering applications. This paper combines the SMM method with the SVSF to create an adaptive formulation of the SVSF. The performance is evaluated and compared with the standard SVSF on an electrohydrostatic actuator (EHA) that was created for experimental analysis.

This paper is organized as follows. Section 2 summarizes the SVSF estimation process. Section 3 introduces the SMM estimator and proposed SMM-SVSF or adaptive SVSF algorithm. Section 4 describes the experimental setup as well as the equations of motion governing the EHA. Section 5 discusses the application of the standard SVSF and adaptive SVSF to the EHA system, followed by concluding remarks.

II. Smooth Variable Structure Filter

The smooth variable structure filter (SVSF) is a predictor-corrector estimation strategy that offers some robustness and stability to disturbances and uncertainties. When an upper bound is defined based on the level of noise and unmodeled dynamics, the SVSF yields a robust estimate to noise, disturbances, and uncertainties [36, 37]. Similar to the KF, the SVSF is model-based and may be applied to both linear or nonlinear systems and measurements [2, 11]. The standard SVSF estimation concept is illustrated in Fig. 1.

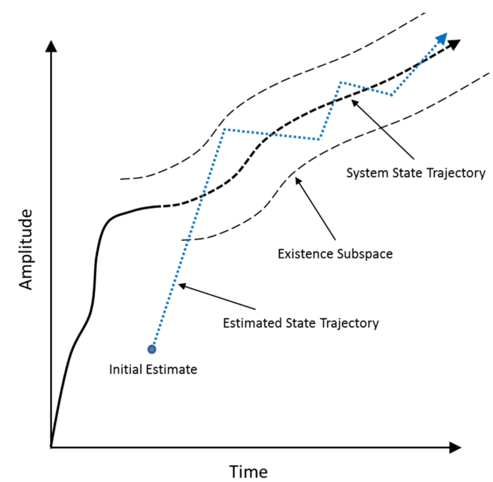

Figure 1. Standard SVS estimation concept with existence subspace boundary layer [2].

As described, the SVSF strategy is structured similarly to the well-known KF, but presents a novel method of gain calculation. As per (2.1) and (2.2), the predicted state (or parameter) estimates $\hat{x}_{k+1|k}$ and state error covariance matrix $P_{k+1|k}$ are first calculated, respectively. The corresponding predicted measurements $\hat{z}_{k+1|k}$ and measurement errors $e_{z,k+1|k}$ are calculated as per (2.3) and (2.4), respectively, using the predicted state estimates $\hat{x}_{k+1|k}$ found in (2.1).

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k \tag{2.1}$$

$$P_{k+1|k} = AP_{k|k}A^T + Q_k \tag{2.2}$$

$$\hat{z}_{k+1|k} = C\hat{x}_{k+1|k} \tag{2.3}$$

$$e_{z,k+1|k} = z_{k+1} - \hat{z}_{k+1|k} \tag{2.4}$$

The gain used by the SVSF is calculated using: 1) the predicted and previously updated measurement errors $e_{z,k+1|k}$ and $e_{z,k|k}$, respectively; 2) the boundary layer widths ψ used to smooth the estimates; 3) and the 'SVSF' convergence rate γ. The gain K_{k+1} is defined by [2]:

$$K_{k+1} = C_k^+ diag\left[\left(\left|e_{z_{k+1|k}}\right| + \gamma\left|e_{z_{k|k}}\right|\right) \circ sat\left(\bar{\psi}^{-1} e_{z_{k+1|k}}\right)\right] \\ \dots diag\left(e_{z_{k+1|k}}\right)^{-1} \quad (2.5)$$

where $\circ$ refers to element-by-element multiplication, and the superscript $+$ refers to the pseudoinverse. The saturation function of (2.5) is defined by:

$$sat\left(\bar{\psi}^{-1} e_{z_{k+1|k}}\right) = \begin{cases} 1, & e_{z_i,k+1|k}/\psi_i \geq 1 \\ \frac{e_{z_i,k+1|k}}{\psi_i}, & -1 < \frac{e_{z_i,k+1|k}}{\psi_i} < 1 \\ -1, & e_{z_i,k+1|k}/\psi_i \leq -1 \end{cases} \quad (2.6)$$

where $\bar{\psi}^{-1}$ is a diagonal matrix based on the smoothing boundary layer ψ for each corresponding measurement, where m is the number of measurements [2]:

$$\bar{\psi}^{-1} = \begin{bmatrix} \frac{1}{\psi_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{1}{\psi_m} \end{bmatrix} \quad (2.7)$$

The state estimates $\hat{x}_{k+1|k}$ and error covariance matrix $P_{k+1|k}$ are respectively updated as per (2.8) and (2.9). Finally, the updated measurement error $e_{z,k+1|k+1}$ is found as per (2.10) and is used in the next iteration.

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} e_{z,k+1|k} \quad (2.8)$$

$$P_{k+1|k+1} = (I - K_{k+1}C)P_{k+1|k}(I - K_{k+1}C)^T + \\ \dots K_{k+1}R_{k+1}K_{k+1}^T \quad (2.9)$$

$$e_{z,k+1|k+1} = z_{k+1} - \hat{z}_{k+1|k+1} \quad (2.10)$$

The existence subspace denoted by the dotted black line shown in Figure 1 refers to the level of uncertainty found in the estimation process, and is typically based on the amount of noise or modeling uncertainties [2]. The existence space β is based on the system and measurement modeling errors, and varies with time [26, 33]. While the width is not precisely known, designer knowledge may be used to define the upper bound. When the smoothing boundary is defined larger than the existence subspace, the estimated states are smoothed. Likewise, if the smoothing term is set too small, chattering (high-frequency switching) may occur due to underestimating the uncertainties.

III. Novel Adaptive Formulation of the SVSF

The static multiple model (SMM) assumes that the system behaves according to a finite number of r models $M^1, M^2, \dots, M^r$. The SMM uses weights μ_k^j at time k for each model M^j in order to combine the corresponding model state estimates [38]. The weights are initially uniformly distributed and subsequent weights are calculated by:

$$\mu_k^j = \frac{p(z_k|M^j)\mu_{k-1}^j}{\sum_{i=1}^r p(z_k|M^i)\mu_{k-1}^i} \quad (3.1)$$

A likelihood value of measurement z_k based on M^j is defined as follows:

$$p(z_k|M^j) = \frac{1}{\sqrt{2\pi\sigma_j^2}} exp\frac{-(z_k - \hat{z}_{k|k-1})^2}{2\sigma_j^2} \quad (3.2)$$

$$\sigma_j^2 = C_k^j P_{k|k-1}^j C_k^{jT} + (\sigma_z^2)^j \quad (3.3)$$

where σ_j^2 refers to the variance of model M^j based on the predicted measurement $\hat{z}_{k|k-1}$ for model M^j [38]. The adaptive estimates are calculated using the weighted sum produced by the system models, as per (3.4).

$$\hat{x}_{k|k} = \sum_{j=1}^r \mu_k^j \hat{x}_{k|k}^j \quad (3.4)$$

The adaptive covariance is calculated in a similar fashion, as shown in (3.5).

$$P_{k|k} = \sum_{j=1}^r \mu_k^j \left[P_{k|k}^j + \left(\hat{x}_{k|k}^j - \hat{x}_{k|k}\right)\left(\hat{x}_{k|k}^j - \hat{x}_{k|k}\right)^T\right] \quad (3.5)$$

The proposed SMM-SVSF (or adaptive SVSF) algorithm uses the model weights from the static multiple models estimator to generate a weighted prediction. The weighted state predictions are used to calculate the SVSF gain which in turn is used to generate an updated state estimate and state error covariance. Since the algorithm uses a weighted combination of system modes, the weights could be used to describe the mixing of different system modes. Figure 2 depicts the algorithm flow chart and Table 1 shows the corresponding pseudocode.

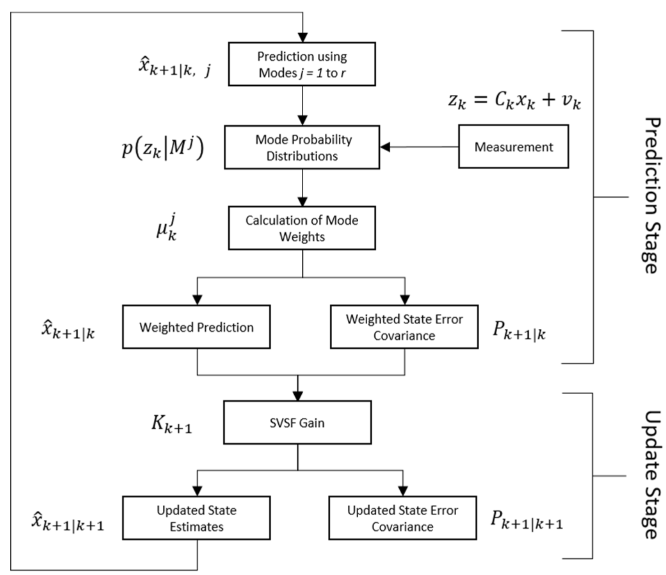

Figure 2. The proposed SMM-SVSF (or adaptive SVSF) algorithm flowchart.

TABLE I. PSEUDOCODE FOR THE SMM-SVSF ALGORITHM

1:	For models (M^j), $j = 1$ to r
	$\hat{x}_{k+1\|k,\_j} \leftarrow (A_j, u)$
2:	For models (M^j), $j = 1$ to r
	$\sigma \leftarrow (Q, R, P_{k\|k})$
	$p \leftarrow (\hat{x}_{k+1\|k,j}, z, \sigma)$
3:	$\mu_{k+1} \leftarrow (\hat{x}_{k+1\|k,j}, \mu_k)$
4a:	$\hat{x}_{k+1\|k} \leftarrow (\hat{x}_{k+1\|k,j}, \mu_{k+1})$
4b:	For operating modes $j = 1$ to r
	$P_{k+1\|k,j} \leftarrow (A_j, \hat{x}_{k+1\|k})$
5:	$P_{k+1\|k} \leftarrow (P_{k+1\|k,j}, \mu_{k+1})$
6:	$K_{k+1} \leftarrow (C, \gamma, \text{saturation})$
7a:	$\hat{x}_{k+1\|k+1} \leftarrow (\hat{x}_{k+1\|k}, z, C, K_{k+1})$
7b:	$P_{k+1\|k+1} \leftarrow (P_{k+1\|k}, C, K_{k+1}, R)$

After the SVSF boundary layer vector and convergence rate have been set and model weights have been initialized, a predicted state estimate for each system model is made. The standard deviation is calculated using three different covariance matrices based on: the state error, the system noise, and the measurement. Next, the updated estimates, standard deviations, and sensor measurements are used to calculate the model probabilities. These probabilities are then used to update the model weights which are used to generate a weighted predicted state estimate and error covariance. This information is fed through the SVSF update stage as described in Section 2 using (2.8) through (2.10).

IV. SIMULATION SETUP AND RESULTS

The electrohydrostatic actuator (EHA) is a type of aerospace actuator used for control of flight surfaces [2]. The EHA can be modelled using four states: the actuator position $x_1 = x$, velocity $x_2 = \dot{x}$, acceleration $x_3 = \ddot{x}$, and differential pressure across the actuator $x_4 = P_1 - P_2$. The physical modeling approach was used to obtain the nonlinear state-space equations in discrete-time described by [2, 43]:

$$x_{1,k+1} = x_{1,k} + T x_{2,k} \quad (4.1)$$

$$x_{2,k+1} = x_{2,k} + T x_{3,k} \quad (4.2)$$

$$x_{3,k+1} = 1 - \left[T\frac{a_2 V_0 + M\beta_e L}{MV_0}\right] x_{3,k} - T\frac{(A_E{}^2 + a_2 L)\beta_e}{MV_0} x_{2,k} \ldots$$
$$\ldots - T\frac{2a_1 V_0 x_{2,k} x_{3,k} + \beta_e L(a_1 x_{2,k}{}^2 + a_3)}{MV_0} sgn(x_{2,k})$$
$$\ldots + T\frac{A_E \beta_e}{MV_0} u \quad (4.3)$$

$$x_{4,k+1} = \frac{a_2}{A_E} x_{2,k} + \frac{(a_1 x_{2,k}{}^2 + a_3)}{A_E} sgn(x_{2,k}) + \frac{M}{A_E} x_{3,k} \quad (4.4)$$

The system input is defined as follows:

$$u = D_p \omega_p - sgn(P_1 - P_2) Q_{L0} \quad (4.5)$$

where ω_p is the pump speed. The definitions and numeric values of the parameters in the state space equations are found in [2].

The friction was modeled using a quadratic function based on the actuator velocity. The friction coefficients were obtained by preforming experiments ranging from 15.6 to 109 radians per second with each data set containing four trials for repeatability [39]. The state estimates were initialized to zero and the covariance matrices for system and measurement noises were defined respectively as $Q = 10^{-9} I_{4x4}$ and $R = 10^{-6} I_{4x4}$, where I is an identity matrix. Furthermore, the state error covariance matrix P was initialized as $10Q$.

Leakage faults were introduced to investigate the effects of parametric uncertainties in the system. The purpose of this study was to demonstrate the efficacy of the proposed SVSF-SMM strategy (presented in Section 3) compared with the standard SVSF defined earlier in Section 2. The SVSF-SMM algorithm demonstrates robustness in the presence of multiple operating modes. Multiple system modes are introduced to the system in the form of leakage faults. In order to obtain the coefficients of the leakage values, the EHA was operated with a constant pump speed of 94.25 radians per second under a series of differential pressures. The differential pressure was modified using a throttling valve in the hydraulic system. To ensure repeatability, five sets of measurements were made. A linear regression was performed on each data set, and the slope and intercept were used to define L and Q_{L0}, respectively. The leakage coefficients and flow rate offsets used for this study are found in [40]. A minor leakage is introduced to the system at $t = 3\,sec$ and a major leakage is introduced at $t = 6\,sec$. The effect on the input flow rate can be seen in Figure 3.

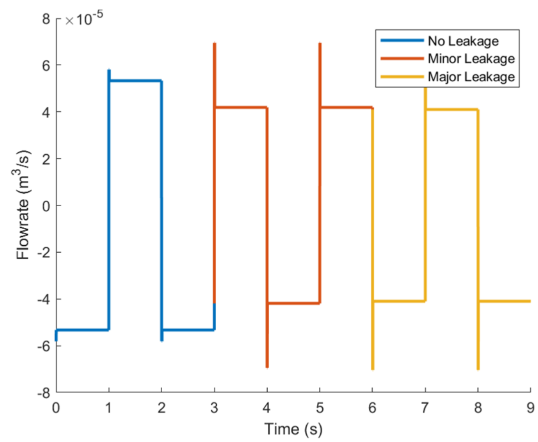

Figure 3. Input flow rate due to internal leakage faults.

The following compares the SVSF and SMM-SVSF in the presence of leakage faults. For the position estimates, the SMM-SVSF performs slightly better than the classical SVSF when the major leakage fault is introduced. The greatest improvement can be seen in the velocity and acceleration estimates. The SVSF filter shows a significant deviation from the true velocity when the minor leakage fault is introduced at 3 seconds, as shown in Figures 4 and 5. The error is exacerbated when the major leakage is introduced at 6 seconds. This error is caused by the modeling uncertainty of the acceleration state, particularly due to flow rate offset of the input. Overall, the SMM-SVSF greatly outperforms the classical SVSF in the presence of modeling uncertainties such as leakage faults. This is expected given that the SMM-SVSF is an adaptive form of the SVSF

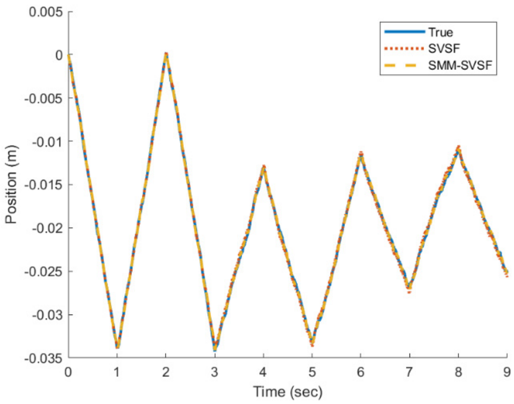

Figure 4. Position estimates for EHA with leakage faults.

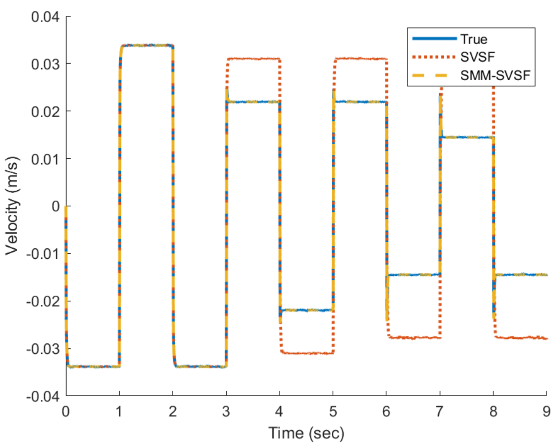

Figure 5. Velocity estimates for EHA with leakage faults.

The SMM-SVSF's ability to determine system modes can be seen in Figure 6 which shows the weights of each system mode used to calculate the estimate. Throughout the entire experiment, the SMM-SVSF filter calculates at least an 80% probability of the correct operating mode at every stage of operation. The figure shows clear transitions from normal operation, to minor leakage, to major leakage at 3 seconds and 6 seconds respectively. The error of the classical SVSF increases during the introduction of faults and spikes when the actuator changes direction. In addition, the RMSE values in Table II show that the SMM-SVSF significantly reduces position, velocity, and acceleration estimation error.

TABLE II. RMSE RESULTS FOR SVSF AND SMM-SVSF

Filter	Position (m)	Velocity (m/s)	Accel. (m/s^2)	Diff. Pres. (Pa)
SVSF	0.0003101	0.0091966	0.002810	0.001002
SMM-SVSF	0.0001828	0.0000799	0.000712	0.001002

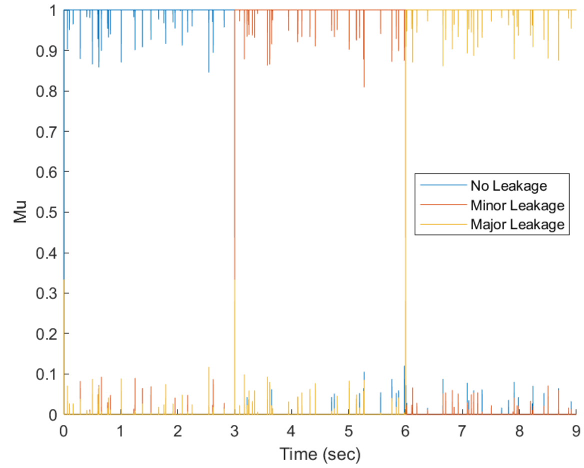

Figure 6. Model probability weights.

CONCLUSIONS

This paper introduced the combination of the SVSF and SMM estimation strategies to create an adaptive SVSF method. A background on estimation theory was provided including a discussion on sliding mode techniques, of which the SVSF is defined. The SMM-SVSF algorithm was detailed in Section 3 and applied to an EHA in order to compare the classical SVSF to the SMM-SVSF. The SMM-SVSF performs well for this particular EHA model due to two main factors: the system parameters of the different leakage modes vary significantly enough for mode differentiation using the SMM method, and the system and measurement noise covariances are well-known. In this case, there is minimal overlap between the predicted probability distributions of each leakage mode which allows for a clear distinction of operating modes. This paper demonstrates that the addition of SMM to the SVSF strategy improves overall estimation process for a system with multiple operating modes, and thereby creates an adaptive SVSF. Potential future work will incorporate additional operating modes such as friction faults as well as the mixing of several different operating modes.

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