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Mass and Compliance Minimization of 3D Printed Polymers

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***Abstract*—The present study compares the results of compliance and mass minimization problems for 3D printed polymers produced using the fused filament fabrication method. Optimization is performed for both the topology and the fiber angle orientation. The compliance minimization optimum is used as the compliance threshold of the mass minimization problem.**

***Index Terms*—Topology optimization, additive manufacturing, compliance minimization, mass minimization**

I. Introduction

Combining topology and print direction optimization leads to unprecedented improvements in the mechanical properties of 3D printed thermoplastic polymers when their optimal parameters are properly selected. Topology optimization (TO) helps to increase the mass savings of a 3D printed part [1], thereby lowering production costs and other costs associated with fabricating a lightweight part. Furthermore, optimizing the print orientation of a printed part offers a unique opportunity to increase the stiffness of the printed geometry, which can help TO save additional mass so that it becomes a closed-loop design process [2, 3]. A massive amount of research has been conducted in the fields of both TO and print direction optimization. However, few studies have performed both optimization techniques simultaneously [4–7]. Even popular simulation commercial software such as Ansys, Abaqus, Nastran, HyperMesh, and Fusion 360 have yet to develop a tool that couples the optimization of both topology and print orientation.

In the present study, we build on previous work done in this field by developing an in-house code that we refer to as "TPO", or topology and print (direction) optimization, to tackle mass and compliance minimization problems while using the method of moving asymptotes (MMA) [8]. This method is proven to have good robustness of the optimization process because of its guaranteed convergence and ease of defining new design variables [9], which we demonstrate in this work. Moreover, the capability of MMA to handle a large number of constraints is essential for future modifications whenever needed.

Adding to that, we focus on the methodology of solving mass and compliance minimization problems for the 3D printed polymers and showing the differences between the results. The modified solid isotropic material with penalization (SIMP), a density-based approach, is used. The results of the proposed TPO method showed a significant increase in the stiffness of the optimized problems, more than solving either topology optimization or print direction optimization. Acrylonitrile butadiene styrene (ABS), a polymer that is widely used in fused filament fabrication, was selected when setting up the optimization problem. It is worth noting that since fiber orientation is determined by the print direction in fused filament fabrication, we alternately used of these two terms to describe the angle.

II. Mathematical formulation

A. Problem statement

The basic compliance minimization problem presented here uses a linear volume fraction constraint and bound constraints on the density variables and the angles, which are the main design variables. The problem statement is as follows:

$$\begin{aligned} \min_{\boldsymbol{\rho},\,\boldsymbol{\theta}} \quad & C(\boldsymbol{\rho},\boldsymbol{\theta}) = \mathbf{U}^T\mathbf{K}(\boldsymbol{\rho},\boldsymbol{\theta})\mathbf{U}. \\ \text{s. t.} \quad & \sum_{i=1}^{N} V_i/V_0 \le V_f, \\ & \mathbf{K}(\boldsymbol{\rho},\boldsymbol{\theta})\mathbf{U} = \mathbf{F}, \\ & \mathbf{0} \le \boldsymbol{\rho} \le \mathbf{1}, \\ & -\pi \le \boldsymbol{\theta} \le \pi. \end{aligned} \tag{1}$$

while the mass minimization problem is stated as follows:

$$\begin{aligned} \min_{\boldsymbol{\rho},\,\boldsymbol{\theta}} \quad & M(\boldsymbol{\rho},\boldsymbol{\theta}) = \sum_{e=1}^{N} \rho_e. \\ \text{s. t.} \quad & C(\boldsymbol{\rho},\boldsymbol{\theta}) \le C_{max}, \\ & \mathbf{K}(\boldsymbol{\rho},\boldsymbol{\theta})\mathbf{U} = \mathbf{F}, \\ & \mathbf{0} \le \boldsymbol{\rho} \le \mathbf{1}, \\ & -\pi \le \boldsymbol{\theta} \le \pi. \end{aligned} \tag{2}$$

where $\boldsymbol{\rho}$ is a vector representing the density of each finite element, $\boldsymbol{\theta}$ is a vector representing the angle of print direction in each finite element, C is the compliance of the structure, $\mathbf{U}$ is the global displacement vector, $\mathbf{K}$ is the global stiffness matrix, N is the number of finite elements, V_i is the volume of an element i, V_0 is the volume of the original design domain,

V_f is the prescribed volume fraction, $\mathbf{F}$ is the global force vector, M is the mass of the structure, ρ_e is the density of element e, and C_{max} is an upper bound on the compliance obtained from the compliance minimization problem.

The solution steps are divided into the following four steps: (1) material model, (2) sensitivity analysis, (3) optimization method, and (4) filtering technique. For details about steps 1, 2, and 4, refer to the work by Andreassen et al. [10]. Only the penalization step is discussed in detail here in order to show how fiber angle orientation is implemented in the problem.

B. Penalization

The SIMP approach assumes isotropic material properties, and hence it is safe to extract the density variable outside the constitutive matrix and use it as a scaling parameter. The main hurdle with performing topology optimization on a composite material is the recalculation of the constitutive matrix (and its sensitivity) for different density values. This is mainly because the constitutive matrix of composite material does not contain the density variable as an explicit parameter that can be extracted outside the matrix. Hence, historically speaking, homogenization methods have been implemented to correctly overcome this obstacle [11]. However, recently, a number of research works have modified the SIMP approach by applying it directly to the constitutive matrix of composite materials. This is the approach we implemented here.

We begin with the stiffness matrix of a 2D finite element with unity thickness as follows [12]:

$$\mathbf{k} = \mathbf{B}^T\mathbf{C}\mathbf{B}. \tag{3}$$

where $\mathbf{B}$ is the strain–displacement matrix, and $\mathbf{C}$ is the constitutive matrix defined as follows for composite material with a density variable:

$$\mathbf{C} = \begin{bmatrix} \frac{\rho^p\, E_1}{1-\nu_{12}} & \frac{\rho^p\, E_2\, \nu_{12}}{1-\nu_{12}^2} & 0 \\ \frac{\rho^p\, E_2\, \nu_{12}}{1-\nu_{12}^2} & \frac{\rho^p\, E_2}{1-\nu_{12}} & 0 \\ 0 & 0 & \rho^p\, G_{12} \end{bmatrix}. \tag{4}$$

The print direction angle is added as a design variable as follows [13]:

$$\boldsymbol{\sigma} = \mathbf{C}'\boldsymbol{\epsilon}, \tag{5}$$

$$\mathbf{C}'(\theta_e) = \mathbf{T_1}(\theta_e)^{-1}\mathbf{C}\mathbf{T_2}(\theta_e), \tag{6}$$

$$\mathbf{T_1} = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & c^2 - s^2 \end{bmatrix}, \tag{7}$$

$$\mathbf{T_2} = \begin{bmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & 2cs & c^2 - s^2 \end{bmatrix}. \tag{8}$$

where c and s represent $cos(\theta)$ and $sin(\theta)$, respectively. According to the classical laminate theory:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \mathbf{T_1}^{-1}\mathbf{C}'\mathbf{T_2} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{bmatrix}, \tag{9}$$

$$\mathbf{k}(\rho_e, \theta_e) = \mathbf{B}^T\mathbf{C}'(\rho_e, \theta_e)\mathbf{B}. \tag{10}$$

C. Optimization method

Since compliance minimization is considered a non-linear programming problem, it can be solved using any of the famous optimization methods, such as Newton or quasi-Newton. However, owing to the large number of design variables being considered here, MMA has already proven to be well suited for solving such problems [8]. The main idea of this method is instead solving (1), which is non-convex. It is replaced by a certain convex function $\widetilde{f}_i(\mathbf{x})$ that can be easily optimized. This convex function is chosen based on the lower and upper asymptotes L_i and U_i, respectively, and the gradient information. Moreover, it is globally convergent and guarantees convergence as long as a feasible solution exists. For details on how the convex functions are derived, refer to [8], the inventor of this code.

III. Applications

Our TPO was developed to help improve the 3D printing of polymers. Thus, the application here focuses on optimizing a 3D-printed ABS layer with a thickness of 0.317 mm. The mechanical properties of the layer were retrieved from the work of Somireddy et al. [14]. The application design space is shown in Fig. 1. A force of 1000 N was applied in both the mass and compliance minimization problems. A compliance minimization problem with a volume fraction of 0.3 is solved first; the minimum compliance calculated was used as the compliance threshold of the subsequent mass minimization problem. For the compliance problem, because of the low volume fraction, the structural members are considered to be carrying only tensile or compressive loads (i.e., a truss-like structure). As a result, all the fibers are oriented in a direction aligned with the load carried by each element. The compliance of the optimized structure is 0.1082.

Figure 3 shows that the compliance nearly reached the minimum after 30 iterations; however, the code had to continue in the optimization process because there were fine changes happening in the densities and angles of each element; these changes can further enhance the compliance value. The changes continued decreasing until a predefined tolerance was reached. The changes in density and angle can be seen in Figs. 4 and 5.

As for the mass minimization problem, the compliance limit was set to 0.1082, which is the minimum reached in the first problem. At the end of the optimization process, the mass reached was 31% of the original mass. The final topology and fiber orientation are shown in Fig. 6. As shown in Fig. 7, the objective function zigzags and reaches the optimum after close to 1200 iterations. This convergence scheme is similar to the steepest descent method convergence, which is linear and incredibly time consuming. Unlike the compliance minimization, changes in angles and densities reached low values, as shown in Figs. 8 and 9, while the mass continued decreasing until it reached 31 % at convergence. As shown, both problems reached nearly the same final topology with

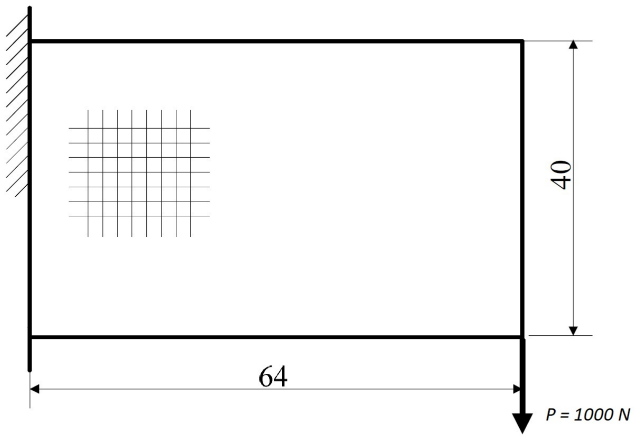

Figure 1. Cantilever beam half fixed from the left and showing the design space dimensions and load.

only minor differences. The advantage of the compliance minimization problem was that the minimum was reached much faster than it was in the mass minimization problem, with no intermediate elements. However, looking at the final topology of both methods, the elements in the compliance minimization problem resulted in a structure with sharper corners, meaning that the corners were not as smooth as those produced in the mass minimization problem. This result favours the mass minimization method over the compliance minimization method, as sharp corners lead to the initiation of stress concentrations in the structure.

Table I
MECHANICAL PROPERTIES OF ABS MATERIAL.

Property	Value
E_1	1775 MPa
E_2	1600 MPa
G_{12}	625 MPa
v_{12}	0.37

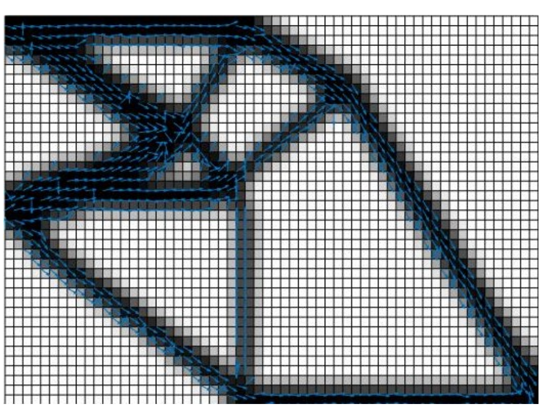

Figure 2. Final topology and fiber orientation after solving the compliance minimization problem.

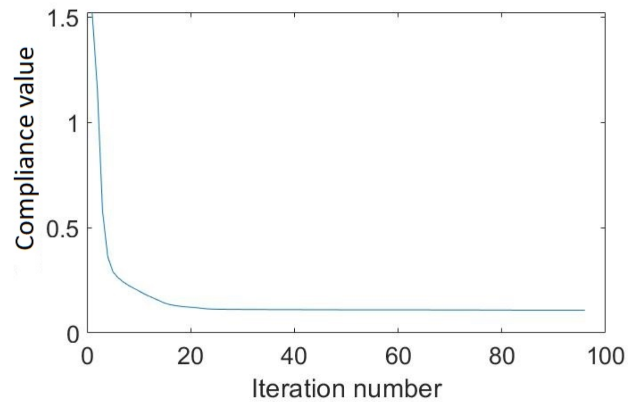

Figure 3. Convergence curve for the compliance minimization problem.

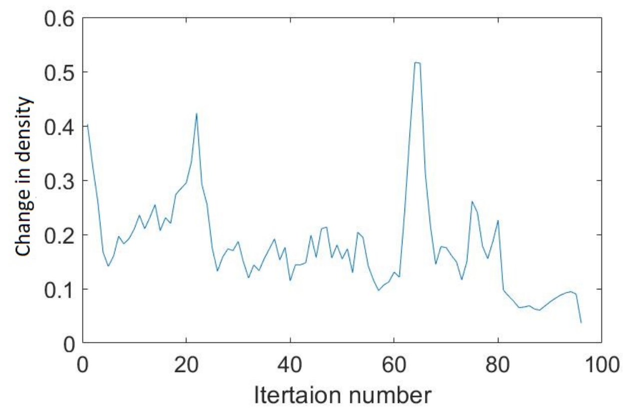

Figure 4. Change in ρ for the compliance minimization problem.

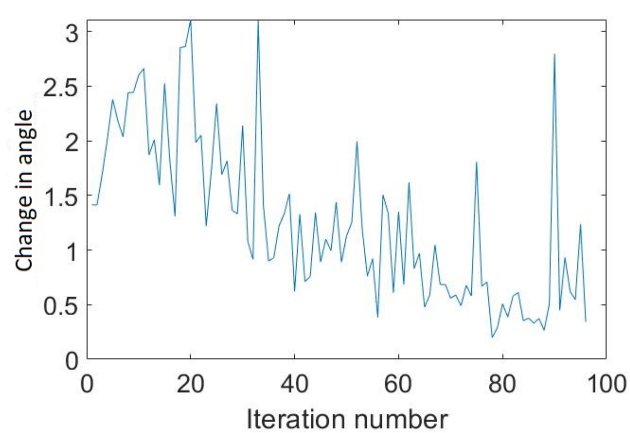

Figure 5. Change in θ for the compliance minimization problem.

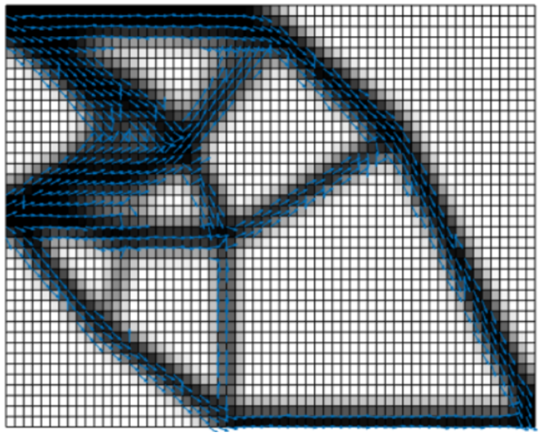

Figure 6. Final topology and fiber orientation after solving the mass minimization problem.

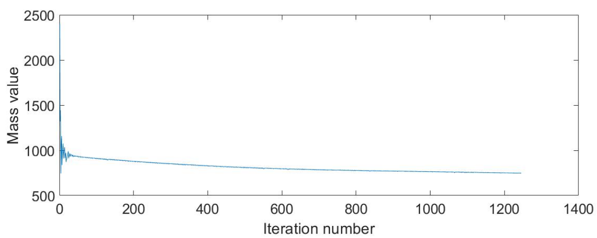

Figure 7. Convergence curve for the mass minimization problem.

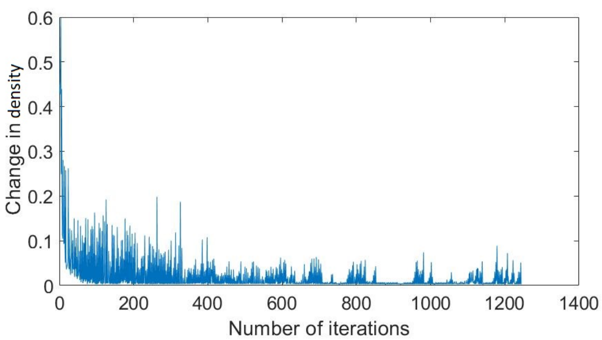

Figure 8. Change in ρ for the mass minimization problem.

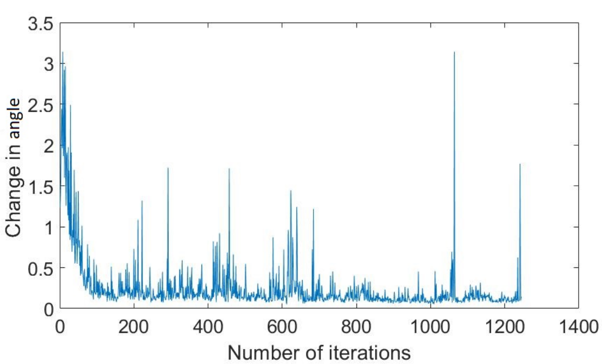

Figure 9. Change in θ for the mass minimization problem.

IV. Conclusion

The main contribution of this paper is that it demonstrates the difference between using mass and compliance as the objective function in the optimization of composites. Our findings were similar whether the aim was mass minimization (with a compliance constraint) or compliance minimization (with a volume constraint). The end result of the compliance problem was the constraint of the mass problem. The MMA method was efficient at handling all the problems without diverging. Adding the fiber angle as a factor in the problem resulted in lower compliance and mass structures, which helped to increase mass savings and hence cost savings. The mass minimization problem resulted in smoother topology elements, while compliance minimization converged far more quickly.

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