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AUTOMATIC DIFFERENTIATION IN THE AUTOMATIC GENERATION OF THE LINEARIZED EQUATIONS OF MOTION

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ABSTRACT

In the ongoing search for mathematically efficient methods of predicting and characterizing the motion of vehicle and other multibody systems, one of the avenues of continued interest is the linearization of the equations of motion. While linearization can potentially result in reduced fidelity in the model, the benefits often make it the pragmatic choice. Linearization also opens other avenues of study, such as modal analysis, frequency response, and linear quadratic control. This work relates to the ongoing development of a multibody dynamics vehicle motion simulation, based on the equations of motion generator code EoM, developed by the University of Windsor Vehicle Dynamics and Control research group, although the results would be equally applicable in any similar implementation. The EoM software can automatically generate the equations of motion for complex three-dimensional multibody systems but restricts the result to linear equations.

This paper explores the application of a technique known as *automatic differentiation* to the automatic generation of the linearized equations of motion. It is important to note that automatic differentiation is distinct from both symbolic differentiation and numerical differentiation (finite differences). Automatic differentiation allows one to numerically evaluate the derivative of any function, with no prior knowledge of the differential relationship to other functions. It exploits the fact that every computer program must evaluate every function using only elementary arithmetic operations. Using automatic differentiation, derivatives of arbitrary order can be computed, accurately to working precision, with minimal additional computational cost over the evaluation of the base function.

This paper focuses on the JuliaDiff software library for implementation of the automatic differentiation tools. Julia is a relatively new programming language that focuses on high-speed numerical computing and offers some advantages over other traditional languages. Several example problems are solved to illustrate the process. The results show that automatic differentiation is a feasible means of generating the appropriate matrix terms in the linearized equations of motion. A comparison with the comparable algebraic formulation shows that automatic differentiation can be surprisingly efficient.