Dimensional Synthesis of a 2-PSS/U Manipulator

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Abstract— **Dimensional synthesis of a parallel kinematic manipulator (PKM) is an optimization exercise wherein the objective function is composed of desirable kinematic performance metrics (e.g., workspace volume, dexterity, stiffness, etc.) and the parameter space is defined by the geometry of the PKM. This paper presents a dimensional synthesis exercise for a 2-PSS/U PKM. To this end, the direct kinematics and the differential kinematics of the PKM was presented and the corresponding singularity configurations were discussed. Finally, a parameter search method was employed to determine a favorable geometry that was later employed to construct a fully functioning physical prototype.**

I. Introduction

High speed manipulation of optical detectors and sensors (e.g., laser projectors, cameras, mirrors) is required for many optomechatronic applications; examples include laser scanning [1], beam steering [2], image stabilization [3], camera orientation [4], tracking [5], free space optical communication [6], etc. Although galvanometer mirrors have been used in beam steering applications (e.g., [7]), their range of motion is generally limited. Alternatively, many optomechatronic applications utilize kinematic mechanisms for orientating optical payloads (i.e., *orientation manipulation*).

An orientation manipulator constraints its payload to only spherical motion [8, p. 28] about a fixed point. In terms of kinematic topology, such manipulators are characterized either by a serial architecture or a parallel architecture. The classic Gimbal mechanism [9], [10] is the most intuitive and the most common embodiment of a serial orientation manipulator. In addition, many parallel kinematic architectures (PKM) featuring three and two rotational degrees of freedom (DOF) have been reported in the literature [11]–[15]. Since the moving platform in a parallel architecture is actuated by multiple kinematic chains as opposed to a single kinematic chain in a serial architecture, the kinematic structure of a PKM is generally more conducive to better performance. Despite the potential advantages of speed, accuracy and stiffness, the limiting factors that may deter the performance of a PKM include workspace volume, presence of multiple singularities in the workspace, limited range of the link lengths, range of the available motion of the joints, and possible link interference [16], [17]. Dimensional synthesis of a parallel manipulator refers to the systematic determination of the optimal geometry (e.g., link lengths and positions of kinematic joints) that minimizes these limitations so that a set of application relevant kinematic performance characteristics can be achieved. In this paper dimensional synthesis of a 2-PSS/U PKM is presented.

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II. Problem Formulation

The exercise of dimensional synthesis of a PKM aims to optimize its kinematic performance characteristics as a function of its geometry. In order to formulate the this optimization exercise for the 2-PSS/U manipulator, the parameter space is defined by the geometric representation of the manipulator and the objective function is composed of desired kinematic performance characteristics.

A. Geometric Parameterization

The kinematic structure of the 2-PSS/U manipulator is presented in Fig. 1. The universal joint that constraints the moving platform to the mechanical ground is decomposed into two revolute joints that are defined by their respective joint axes $\hat{\mathbf{w}}_i$ $(i = 1, 2)$. The joint axes $\hat{\mathbf{w}}_i$ are not constrained in any way except that they intersect at the mechanism center O, and they are perpendicular to each other. The link $\mathrm{A}_i\mathrm{B}_i$ is a linearly extensible limb (i.e., prismatic joint), which is also the actuated joint. Furthermore, the articulation points B_i and C_i refer to the locations of the spherical joints. The link $\mathrm{C}_1\mathrm{C}_2$ constitutes the moving platform of the manipulator.

The geometry of the PKM is defined in the following manner. When the two identical prismatic actuators are at mid-stroke, the manipulator is defined to be at its home position. In addition, both actuators are constrained to operate in the vertical direction. At the home position, points A_1 and A_2 lie in a horizontal plane. The moving platform articulation points C_1 and C_2 also lie in a different horizontal plane at the home position. The revolute joint axis $\hat{\mathbf{w}}_2$ lies in the plane defined by the points A_1, B_1 and C_1 at the home position. Correspondingly, the axis $\hat{\mathbf{w}}_1$ is coplanar with the plane defined by the points A_2, B_2 and C_2 at the home position. Finally, each of the point pairs (C_1, C_2) and (A_1, A_2) are constrained to be equidistant from the mechanism center O. Under these constraints, the geometry of the manipulator can be defined by four parameters (r, R, h, H). At the home position, r and R are the horizontal distances between the

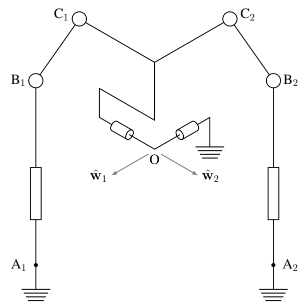


Fig. 1. Kinematic structure of the 2-$\underline{\text{P}}$SS/U parallel orientation manipulator.

points O and C_i and B_i respectively. In addition, h and H are the vertical distances between the points O and C_i and B_i respectively. It should be noted that the aforementioned geometric configuration does not allow decoupling of the two degrees of freedom.

B. Objective Function

PKM performance characteristics that are commonly studied in the related literature include workspace volume [18], dexterity [19], accuracy [20], stiffness [21], etc. In this paper, workspace volume and dexterity have been selected as the two desirable kinematic performance metrics. Tilt & Torsion (T&T) angles [22] provide a convenient parametrization of the 2D workspace volume. Since the 2-$\underline{\text{P}}$SS/U manipulator is characterized as torsion-restricted, the tilt and the azimuth angles can sufficiently define the 2D workspace. The *reachable* workspace is defined as all the tilt angles that the manipulator can reach with *some* azimuth angle, and the *regular* workspace is defined as all the tilt angles that the manipulator can reach with *any* azimuth angle. The maximum tilt angle θ_T is chosen to represent the volume of the regular workspace in this paper.

A frequently cited dexterity index is the reciprocal of the Euclidean norm condition number of the inverse Jacobian matrix (e.g., [16]), which measures only the local dexterity of the point at which the inverse Jacobian is evaluated. The quality of the entire workspace can be quantified by the global conditioning index (GCI) [23], which is an integral of the local dexterity index over the entire workspace. Although GCI provides an aggregated measure of the dexterity characteristics of a workspace, one of the drawbacks arises from its inability to indicate any poor local behavior [24]. However, this limitation is mitigated by considering the minimum local dexterity index of a workspace as an additional kinematic performance metric.

The GCI, the minimum dexterity d_m and the maximum tilt angle θ_T provide a multi-dimensional objective function for the dimensional synthesis problem in this paper.

III. Kinematic Analysis

The kinematic model of the 2-$\underline{\text{P}}$SS/U architecture has been analyzed in [15], [25]. It should be mentioned that the inverse model is very similar to that of the 3-$\underline{\text{P}}$SS/S architecture. However, a closed-form solution to the direct kinematics of the 2-$\underline{\text{P}}$SS/U architecture is not found in the literature. The iterative solution to the direct kinematics problem provided in [15] numerically estimates one of the two workspace coordinates, and the remaining coordinate is analytically determined in a subsequent step. In contrast, this paper formulates the direct kinematics problem as an exactly defined system of nonlinear equations, which is iteratively solved to obtain both workspace coordinates simultaneously. In order to facilitate the formulation of the direct kinematics problem, two coordinate frames are established first. Let the coordinate frame A be fixed in space (i.e., inertial frame) and the coordinate frame B (i.e., body-fixed frame) be embedded in the moving platform. The origins of these two coordinate frames are coincident at the mechanism center O. Since the position vectors of the articulation points (i.e., terminal points of the links) can be defined with respect to either coordinate frame, it is necessary to unambiguously specify the coordinate frame in which a vector is referenced. To this end, a preceding superscript is used to denote the coordinate frame in which a vector is expressed; e.g., ${}^{\mathrm{x}}\mathbf{a}$ denotes the vector $\mathbf{a}$ expressed in the X coordinate frame. The rotation matrix that defines the relative orientation of the Y coordinate frame with respect to the X coordinate frame is denoted by ${}^{\mathrm{x}}\mathbf{R}_{\mathrm{y}}$. If a vector is denoted by ${}^{\mathrm{x}}\mathbf{a}$ and ${}^{\mathrm{y}}\mathbf{a}$ with respect to two separate coordinate frames, the rotation matrix ${}^{\mathrm{x}}\mathbf{R}_{\mathrm{y}}$ provides the following transformation: ${}^{\mathrm{x}}\mathbf{a} = {}^{\mathrm{x}}\mathbf{R}_{\mathrm{y}} \times {}^{\mathrm{y}}\mathbf{a}$.

A. Direct Kinematics

Without loss of any generality, let the x axis of the inertial frame A point along $\hat{\mathbf{w}}_1$ and the y axis point along $\hat{\mathbf{w}}_2$ at the home position of the manipulator. The z axis of the inertial frame A is determined by the right hand rule. In order to define the orientation of the body-fixed frame B, it is assumed that frames A and B coincide initially. The final orientation of the frame B is reached by rotating the body-fixed frame about the x axis of the inertial frame A by an angle ψ in a first rotation. A second rotation of the body-fixed frame about the rotated y axis (i.e., $\hat{\mathbf{w}}_2$ axis) of the frame B by an angle ϕ provides the final orientation of frame B. Following the aforementioned Euler angles convention, the rotation matrix ${}^{\mathrm{a}}\mathbf{R}_{\mathrm{b}}$ is provided by,

$$
{}^{\mathrm{a}}\mathbf{R}_{\mathrm{b}} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ \sin\psi\sin\phi & \cos\psi & -\sin\psi\cos\phi \\ -\cos\psi\sin\phi & \sin\psi & \cos\psi\cos\phi \end{bmatrix}. \quad (1)
$$

Geometry of the moving platform provides the position vectors of points C_i with respect to the body-fixed frame B; i.e.,

$$
{}^{\mathrm{b}}\mathbf{c}_i = \begin{bmatrix} c_{iu} & c_{iv} & c_{iw} \end{bmatrix}^T .
$$

In order to obtain ${}^{\mathrm{a}}\mathbf{c}_i$,

$$
{}^{\mathrm{a}}\mathbf{c}_i = {}^{\mathrm{a}}\mathbf{R}_{\mathrm{b}} \times {}^{\mathrm{b}}\mathbf{c}_i. \quad (2)
$$

Expanding (2) yields (3).

$$
{}^{\mathrm{a}}\mathbf{c}_i = \begin{bmatrix} \cos\phi\, c_{iu} + \sin\phi\, c_{iw} \\ \sin\psi\sin\phi\, c_{iu} + \cos\psi\, c_{iv} - \sin\psi\cos\phi\, c_{iw} \\ -\cos\psi\sin\phi\, c_{iu} + \sin\psi\, c_{iv} + \cos\psi\cos\phi\, c_{iw} \end{bmatrix} \quad (3)
$$

Let, $\overline{B_iC_i} = \mathbf{x}_i$, $\|\mathbf{x}_i\| = x_i$, $\|\mathbf{b}_i\| = b_i$, and $\|\mathbf{c}_i\| = c_i$. Since $\mathbf{x}_i = \mathbf{b}_i - \mathbf{c}_i$, it can be written that $\|\mathbf{b}_i - \mathbf{c}_i\|^2 = x_i^2$. The eqaution in (4) can be written by expressing the vectors in coordinate frame A.

$$f_i := \|{}^{\mathrm{a}}\mathbf{b}_i - {}^{\mathrm{a}}\mathbf{c}_i\|^2 - x_i^2 = 0 \tag{4}$$

Writing (4) for $i = 1, 2$ provides a system of two nonlinear equations; i.e.,

$$F := \begin{bmatrix} \|{}^{\mathrm{a}}\mathbf{b}_1 - {}^{\mathrm{a}}\mathbf{c}_1\|^2 - x_1^2 \\ \|{}^{\mathrm{a}}\mathbf{b}_2 - {}^{\mathrm{a}}\mathbf{c}_2\|^2 - x_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{5}$$

Since ${}^{\mathrm{a}}\mathbf{b}_i$ is known in a direct kinematics problem, and ${}^{\mathrm{a}}\mathbf{c}_i$ can be obtained from (3), F in (5) becomes a system of equations in two unknowns $[\psi \quad \phi]^T$. Solving (5) iteratively for the unknowns ψ and ϕ provides the solution for the direct kinematics problem. A nonlinear least squares analysis can be employed to this end. The corresponding Jacobian matrix can be conveniently obtained from a computer algebra system (CAS).

B. Differential Kinematics

In order to facilitate the study of the differential kinematics, the angular velocity of the link B_iC_i is denoted by $\boldsymbol{\omega}_i$ and the angular velocity of the moving body is denoted by $\boldsymbol{\omega}_c$. In addition, the angular displacements about the axes $\hat{\mathbf{w}}_1$ and $\hat{\mathbf{w}}_2$ are denoted by ψ and ϕ respectively. The linear velocity of point C_i is provided by,

$$\mathbf{v}_{ci} = \boldsymbol{\omega}_c \times \mathbf{c}_i. \tag{6}$$

In terms of the the velocity of the actuated joints (i.e., $\dot{d}_i$), $\mathbf{v}_{ci}$ can also be calculated as,

$$\mathbf{v}_{ci} = \dot{d}_i \hat{\mathbf{n}}_i + \boldsymbol{\omega}_i \times \mathbf{x}_i. \tag{7}$$

Equating (6) and (7) provides,

$$\boldsymbol{\omega}_c \times \mathbf{c}_i = \dot{d}_i \hat{\mathbf{n}}_i + \boldsymbol{\omega}_i \times \mathbf{x}_i. \tag{8}$$

Dot multiplying both sides of (8) by $\mathbf{x}_i$ and subsequent rearranging using the vector triple product rule yields,

$$(\mathbf{c}_i \times \mathbf{x}_i) \cdot \boldsymbol{\omega}_c = (\hat{\mathbf{n}}_i \cdot \mathbf{x}_i) \dot{d}_i. \tag{9}$$

Writing (9) for $i = 1, 2$ provides two scalar equations that can be arranged in matrix format as,

$$\begin{bmatrix} (\mathbf{c}_1 \times \mathbf{x}_1)^T \\ (\mathbf{c}_2 \times \mathbf{x}_2)^T \end{bmatrix} \times \boldsymbol{\omega}_c = \begin{bmatrix} (\hat{\mathbf{n}}_1 \cdot \mathbf{x}_1) & 0 \\ 0 & (\hat{\mathbf{n}}_2 \cdot \mathbf{x}_2) \end{bmatrix} \times \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \end{bmatrix} \tag{10}$$

The angular velocity vector $\boldsymbol{\omega}_c$ can be written as,

$$\begin{aligned} \boldsymbol{\omega}_c &= \dot{\psi} \hat{\mathbf{w}}_1 + \dot{\phi} \hat{\mathbf{w}}_2 \\ &= \begin{bmatrix} \hat{\mathbf{w}}_1 & \hat{\mathbf{w}}_2 \end{bmatrix} \times \begin{bmatrix} \dot{\psi} \\ \dot{\phi} \end{bmatrix}. \end{aligned} \tag{11}$$

Substituting (11) into (10) and subsequent rearranging provides,

$$\begin{aligned} &\begin{bmatrix} (\mathbf{c}_1 \times \mathbf{x}_1) \cdot \hat{\mathbf{w}}_1 & (\mathbf{c}_1 \times \mathbf{x}_1) \cdot \hat{\mathbf{w}}_2 \\ (\mathbf{c}_2 \times \mathbf{x}_2) \cdot \hat{\mathbf{w}}_1 & (\mathbf{c}_2 \times \mathbf{x}_2) \cdot \hat{\mathbf{w}}_2 \end{bmatrix} \times \begin{bmatrix} \dot{\psi} \\ \dot{\phi} \end{bmatrix} \\ &\qquad = \begin{bmatrix} (\hat{\mathbf{n}}_1 \cdot \mathbf{x}_1) & 0 \\ 0 & (\hat{\mathbf{n}}_2 \cdot \mathbf{x}_2) \end{bmatrix} \times \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \end{bmatrix}. \end{aligned} \tag{12}$$

Equation (12) can be rewritten as,

$$J_x \times \begin{bmatrix} \dot{\psi} & \dot{\phi} \end{bmatrix}^T = J_q \times \begin{bmatrix} \dot{d}_1 & \dot{d}_2 \end{bmatrix}^T. \tag{13}$$

The orientation manipulator is said to be in a *singular configuration* when at least one of the two matrices J_x and J_q is singular.

C. Inverse Kinematic Singularities

Since J_q is a diagonal matrix, it is singular only when at least one of the diagonal entries is zero; i.e., $\hat{\mathbf{n}}_i \cdot \mathbf{x}_i = 0$ or $\hat{\mathbf{n}}_i \perp \mathbf{x}_i$. Therefore, when the passive link B_iC_i is perpendicular with the axis of the prismatic joint A_iB_i, the mechanism is in an inverse kinematic singular configuration. When J_q is singular and its null space is not empty, there exist some non-zero $\dot{d}_i$ for which $[\dot{\psi} \quad \dot{\phi}]^T$ is zero; i.e., certain infinitesimal motion of the moving platform at the singular configuration (i.e., $A_iB_i \perp B_iC_i$) cannot be achieved despite the application of actuation forces.

D. Direct Kinematic Singularities

When J_x is singular, there exists some non-zero $[\dot{\psi} \quad \dot{\phi}]^T$ that yields zero $\dot{d}_i$; i.e., even though the actuators are fixed, the moving platform of the mechanism can exhibit infinitesimal motion in some directions.

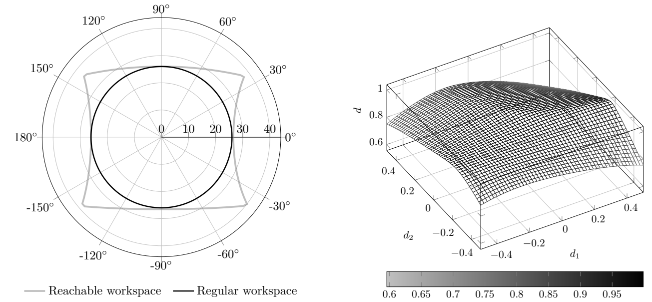
Case 1: When one of the rows of J_x vanishes, it becomes singular. This occurs when $(\mathbf{c}_i \times \mathbf{x}_i)$ lies in the same plane defined by $\hat{\mathbf{w}}_1 \times \hat{\mathbf{w}}_2$. Physically, when points B_i and C_i ($i = 1, 2$) coincides with the plane $\hat{\mathbf{w}}_1 \times \hat{\mathbf{w}}_2$ this direct kinematic singularity occur.

Case 2: The Jacobian matrix J_x becomes deficient in column rank when the four articulation points C_1, C_2, B_1 and B_2 are coplanar with either $\hat{\mathbf{w}}_1$ or $\hat{\mathbf{w}}_2$.

Case 3: Each row of the Jacobian matrix J_x defines a vector in a plane defined by the universal joint axes $\hat{\mathbf{w}}_1$ and $\hat{\mathbf{w}}_2$. When the two row vectors coincide, J_x becomes singular. In geometric terms, when the planes defined by $(\mathbf{c}_1 \times \mathbf{x}_1)$, $(\mathbf{c}_2 \times \mathbf{x}_2)$ and $(\hat{\mathbf{w}}_1 \times \hat{\mathbf{w}}_2)$ intersect on a single line, the mechanism is at a direct kinematic singularity configuration.

IV. Dimensional Synthesis

Since the coupling between the two kinematic loops of the manipulator was observed to be insignificantly small, the motion of each loop can be approximated as an independent spatial slider-crank mechanism of trivial complexity. In order to synthesize the dimensions of this manipulator, two such models must be optimized for kinematic performance. However, application of a sophisticated optimization method was considered to be unnecessary because interaction between the two models was *practically* nonexistent. As an alternative, the parameter space was explored under a Latin hypercube sampling scheme, and the kinematic performances estimated at these sample points were employed to choose the preferred dimensions of the manipulator. Subsequently, the design that was chosen for its high kinematic performance is characterized by the following geometric parameters: $r = 1.1$, $R = 1.6$, $h = 0.2$, and $H = 4.0$. For the sake of computational convenience, the values of these parameters were normalized



(a) Reachable and regular workspaces of the selected design. Radial axis: tilt angle and Angular axis: azimuth angle.

(b) Local dexterity as a function of the actuated joint coordinates (the joint coordinates are normalized over the range ± 0.5).

Fig. 2. Kinematic performance of the torsion-restricted POM.

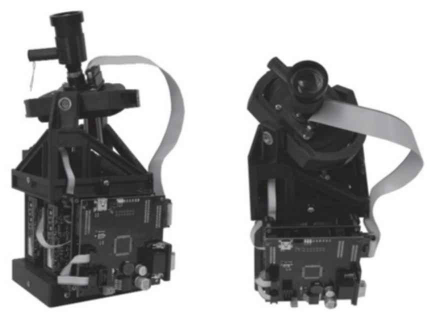


Fig. 3. Prototype implementation of a 2-PSS/U manipulator employing the synthesized geometry.

with respect to the stroke of the actuator. The kinematic performance metrics corresponding to the selected design are graphically presented in Fig. 2. Specifically, these metrics were estimated as GCI = 0.9183, $d_m = 0.6592$, and $\theta_T = 26.0156°$. The physical prototype developed employing the synthesized geometry is shown in Fig. 3.

V. Conclusion

This paper presents a new solution methodology for the direct kinematics problem for the 2-PSS/U PKM. The iterative solution enabled the estimation of kinematic performance metrics such as the GCI, workspace volume and local dexterity as a function of the PKM's geometry. The parameter space defined by the PKM's geometry was explored under a Latin Hypercube sampling scheme to quantify the objective function. Although the direct and the differential kinematics are analytic in nature, the dimensional synthesis problem was solved numerically.

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