

# DEVELOPMENT OF AN ADAPTIVE WEIGHTED PREDICTION-BASED MODEL PREDICTIVE CONTROL USING DISTURBANCE ESTIMATION FOR LONGITUDINAL AUTONOMOUS DRIVING

Kwangseok Oh<sup>1</sup> and Jaho Seo<sup>2\*</sup>

<sup>1</sup>School of ICT, Robotics & Mechanical Engineering, Hankyong National University, Anseong-si, Korea

<sup>2</sup>Department of Automotive and Mechatronics Engineering,

Ontario Tech University, Oshawa, Canada

\*Jaho.Seo@ontariotechu.ca

**Abstract**—This paper presents an adaptive weighted prediction-based model predictive control using disturbance estimation for longitudinal autonomous driving. To reduce the negative effect of uncertainties on the control performance of model predictive control, the weighting function for linear prediction was designed based on an exponential function with disturbance estimation. The main role of the weighting function proposed in this study is to fade the effect of uncertainties as the prediction step increases. The disturbance of the longitudinal control model for autonomous driving was estimated using a sliding mode observer. The estimated disturbance was used to determine the main parameter of the weighting function. Performance evaluations were conducted using commercial software for reasonable evaluations of the control algorithm. Evaluation results show that the weighted prediction-based control algorithm can provide accurate tracking of desired states compared to the constant prediction-based control algorithm.

**Keywords**—*model predictive control; autonomous driving; weighted prediction; sliding mode observer; disturbance estimation*

## I. INTRODUCTION

Model predictive control (MPC) algorithm has been widely used in various industrial fields for optimal control of multi-input multi-output systems with input and state constraints. Generally, the MPC process is divided into three steps: error calculation, derivation of predictive outputs, and optimal input calculation through an optimization process. In the second and third steps, a relatively accurate mathematical model of the system is needed for reasonable prediction and accurate control inputs. However, there always exist unpredictable uncertainties between the mathematical model and the actual system. The uncertainties can be increased or decreased in the prediction step of the MPC formulation and this can have a negative influence on the control performance of the MPC. Therefore, various studies have been conducted to overcome the aforementioned limitation of the inherent MPC algorithm.

Li et al [1] proposed a novel self-triggered MPC algorithm for simultaneous triggering and control using the total cost function that considers communication cost explicitly. They developed theoretical conditions on ensuring feasibility and closed-loop stability for constrained nonlinear systems. Hou [2] developed an adaptive MPC algorithm for load torque estimation and prediction for the electric ship propulsion system that has a challenge of measuring load torque due to multi-frequency fluctuations. To evaluate the proposed MPC's effectiveness, an input observer with linear prediction was developed for comparative study. In [3], an adaptive MPC trajectory tracking system was developed for autonomous wheel loaders, which could deal with the impact of curving paths on the trajectory tracking performance to improve tracking accuracy. Onkol [4] proposed the adaptive MPC algorithm to control the fast-varying error state in the inner loop for a two-wheeled robot manipulator with varying mass. M. Tsao et al. [5] presented a stochastic MPC algorithm that can leverage short-term probabilistic forecasts for dispatching and rebalancing autonomous mobility-on-demand systems. To design this control algorithm, the authors presented the core stochastic optimization problem in terms of a time-expanded network flow model. Seo et al. [6] developed a motion planning algorithm for lane change with a combination of probabilistic and deterministic prediction for automated driving under complex driving circumstances. A collision probability was defined by using a reachable set of uncertainty propagation and the lane change risk was monitored using the predicted time-to-collision and safety distance to guarantee safety in lane change behavior. Moser et al. [7] proposed a stochastic model predictive control approach to optimize the vehicle's fuel consumption. The authors developed a conditional linear Gauss model and trained it with real measurements to estimate the probability distribution of the future velocity. He et al. [8] proposed a stochastic MPC of an air conditioning system to improve the energy efficiency of electric vehicles. In the study, a Markov-chain-based velocity predictor was adopted to provide the states of future disturbances over the stochastic MPC horizon. Also, three control approaches were compared

in terms of electricity consumption, cabin temperature, and comfort fluctuation for reasonable performance evaluation.

In the previous studies, it is found that disturbance estimation and prediction methods have been usually used to design an adaptive MPC with the observer-based estimation and stochastic approach. The estimated value of the current disturbance can be obtained by applying several types of observers or filtering methods. However, accurate prediction of future (potential) disturbances is difficult due to unpredictable changes in internal dynamics and environmental factors. To tackle the problem of inaccurate prediction of disturbance that degrades the MPC control performance, this study proposes an adaptive MPC algorithm for longitudinal autonomous driving by the application of the weighted prediction method using an exponential function. For the weighted prediction of the MPC, the decreasing exponential function was designed to determine the weighting factors at each prediction step. The time constant as a main parameter in the decreasing exponential function was determined using the time-varied disturbance. The proposed MPC was designed such that the time constant decreases to lessen the effect of the predicted far future states when the magnitude of the disturbance change rate is increased. Also, the disturbance of the system model used for the MPC formulation was estimated using a sliding mode observer under finite stability conditions. The algorithm was constructed in Matlab/Simulink environment and its performance evaluation was conducted using the commercial software (CarMaker).

The rest of the paper is organized as follows. Section 2 describes the adaptive weighted prediction-based MPC algorithm for longitudinal autonomous driving. Section 3 presents the evaluation results. Finally, concluding remarks are provided in Section 4 with future works.

## II. ADAPTIVE WEIGHTED PREDICTION-BASED MPC USING DISTURBANCE ESTIMATION

### A. Weighted prediction-based MPC

Figure 1 shows an overall model schematics of the adaptive weighted prediction-based MPC designed in the study.

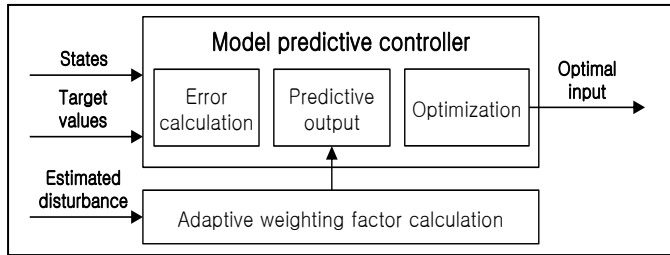


Figure 1. Overall model schematics of the designed MPC algorithm.

In the first step of the MPC process, error states are calculated using the system states and target values defined for tracking control. Then, the predictive outputs are derived using the computed error states and system model during the second step in which the adaptive weighting factors are derived based on the estimated disturbance to compute the predictive outputs. In the final step, optimal control inputs are computed with input/state constraints. The MPC controller was designed based

on a longitudinal kinematic model that represents the kinematic relationship between the subject and preceding vehicles. Figure 2 shows a driving scenario that the subject vehicle is driving with the preceding vehicle.

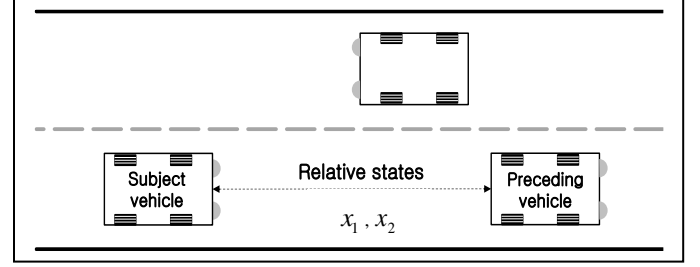


Figure 2. Driving scenario: subject vehicle's driving with the preceding vehicle

In Fig. 2,  $x_1$  and  $x_2$  represents the relative states such as clearance and relative velocity between the subject and preceding vehicles. Using the relative states, the state-space kinematic model can be derived as follows.

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} a_s + \begin{bmatrix} 0 \\ 1 \end{bmatrix} a_p. \quad (1)$$

where  $e_1$  and  $e_2$  represent errors such as clearance error and relative velocity between the subject and preceding vehicles.  $a_s$  and  $a_p$  are each longitudinal acceleration of the subject and preceding vehicles. The longitudinal acceleration of the preceding vehicle was regarded as a disturbance and estimated using a sliding mode observer. The longitudinal acceleration of the subject vehicle is an optimal control input derived from the MPC to minimize the magnitude of error states  $e_1$  and  $e_2$ .

For the computation of the optimal longitudinal acceleration, the discretized state-space equation was derived and predictive error  $y$  was calculated using the weighting matrix  $C$  and Eq. (1). The following equations are the discretized state-space equation, output, and predictive output vector  $Y$ , respectively.

$$e_{k+1} = A_d e_k + B_d u + F_d w_k. \quad (2)$$

$$y_k = C e_k. \quad (3)$$

$$Y = G e_k + H U + F u_k + M W + K w_k. \quad (4)$$

where  $U$  is the optimal input vector and the predictive output vector  $Y$  contains  $N$  predicted output errors. The matrices  $G$ ,  $H$ ,  $F$ ,  $M$ ,  $W$ , and  $K$  were derived using the discretized matrices  $A_d$ ,  $B_d$ ,  $F_d$ , and output matrix  $C$  [9]. Based on the predictive output and the defined parameters that include the input weighting factor  $r$ , output matrix  $C$ , and difference matrix  $D$ , the cost function  $J$  was designed with the adaptive weighting function  $Q_w$ . Equation (5) presents the designed cost function for the MPC input.

$$J = Y^T Q_w^T Q_w Y + r U^T D^T D U. \quad (5)$$

The adaptive weighting function in the cost function was designed using an exponential function with a negative exponent and a finite convergence condition. The following equation  $Q_{w,i}$  is the adaptive weighting function designed in this study, which was used to identify diagonal elements of the weighting matrix  $Q_w$ .

$$Q_{w,i} = \sqrt{e^{-\frac{1}{\tau}(\Delta t \times i)}}, \quad (i = 1, 2, \dots, N). \quad (6)$$

where  $\Delta t$  and  $\tau$  are the discretized time and time constant, respectively. The value of the weighting function varies with the time constant and it has the range of (0,1]. In this study, the time constant was designed using the disturbance (i.e., longitudinal acceleration of the preceding vehicle  $a_p$  in Eq. (1)) to reduce its negative effect on control performance. Also, in our design, the time constant value is decreased to reduce the value of the weighting factor if the magnitude of the disturbance's change rate is increased as the prediction step increase. This relationship between the time constant and the disturbance's change rate is shown in Fig. 3.

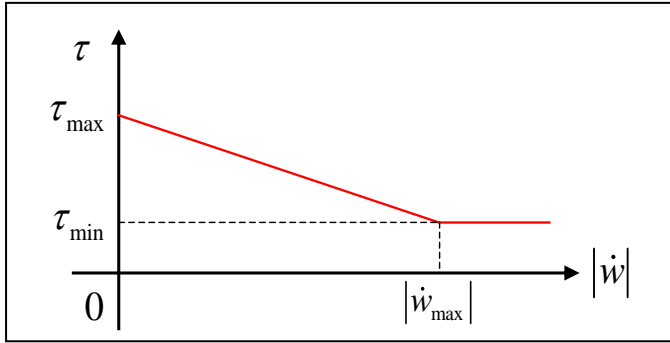


Figure 3. Relationship between the time constant and the disturbance's change rate

Eqs. (7), (8), and (9) are the designed time constant function and its max and min limits, respectively according to the range of disturbance's change rate.

$$\tau_k = \frac{\tau_{\min} - \tau_{\max}}{|\dot{w}_{\max}|} |\dot{w}_k| + \tau_{\max}. \quad (7)$$

$$\text{if } |\dot{w}_k| = 0 \rightarrow \tau_k = \tau_{\max}. \quad (8)$$

$$\text{if } |\dot{w}_k| \geq |\dot{w}_{\max}| \rightarrow \tau_k = \tau_{\min}. \quad (9)$$

where  $\tau_{\min}$ ,  $\tau_{\max}$ , and  $|\dot{w}_{\max}|$  are the minimum and maximum time constant, and the maximum magnitude of disturbance change rate as a design parameter for the time constant function..  $|\dot{w}_k|$  is the current magnitude of the system's disturbance change rate in Eq. (2).

In this study, the disturbance  $a_p$  was estimated using a sliding mode observer and its change rate was also used for the time constant calculation. The next subsection describes the

disturbance estimation algorithm using a sliding mode observer.

### B. Sliding mode observer-based disturbance estimation

The sliding mode observer was designed to estimate the disturbance  $a_p$ . For this design, the observer dynamic model in Eq. (10) was used to derive an error model with the linear transformation. The output  $y$  and transformation matrix  $T$  used for state transformation are shown in Eq. (11) and (12).

$$\dot{\hat{e}} = A_t \hat{e} + B_t a_s + G_n v. \quad (10)$$

$$y = C e. \quad (11)$$

$$T = \begin{bmatrix} \text{Null}(C)^T & C^T \end{bmatrix}^T. \quad (12)$$

where  $A_t$  and  $B_t$  are the matrices for the linearly transformed system and input.  $C$  and  $G_n$  are the output and injection distribution matrices for continuous error dynamics.  $\hat{e}$  and  $v$  are the estimated error and injection term. The matrices  $C$  and  $G_n$  are defined as  $[1 \ 1]$  and  $[L_1 \ L_2]^T$ .  $L$  is the design parameter and  $I$  is the identity matrix. The transformation matrix contains the output matrix and its nullspace for the separation of error states into two states (new error and output error). Using Eqs. (1) and (10), the error dynamics for the observer can be derived with the error state,  $\tilde{e} = \hat{e} - e$  as follows.

$$\dot{\tilde{e}} = A_t \tilde{e} + G_n v. \quad (13)$$

Based on the linear transformation using  $T$ , the error dynamics can be partitioned into the new transformed error state and output error. The injection term,  $v$  was designed for error convergence of the error dynamics in Eq. (13) using the output error and the magnitude  $\rho$  of injection term designed for observer stability.

$$v = -\rho \text{sign}(e_y). \quad (14)$$

With the assumption that the absolute value of the total right side of output error dynamics, Eq. (13) except for the term that contains the injection  $v$  is bounded by value  $L_b$ ,  $\rho$  was designed for error convergence using the Lyapunov direct method with eta-reachability conditions [10] as shown in Eq. (15).

$$\rho = L_b + \eta, \quad \eta > 0. \quad (15)$$

where  $\eta$  is the design parameter that has a strictly positive value to ensure stability margin of the designed sliding mode observer.

The values of  $L_1$  and  $L_2$  of the matrix  $G_n$  were determined by multiplying the transformation matrix and disturbance matrix in Eq. (1) for estimation of the disturbance

after the output error is converged to zero. After the output error terms in Eq. (13) are considered as zero, the eigenvalue of error dynamics in Eq. (13) for the new state error has a negative value based on the determined  $L_1$  and  $L_2$ . Therefore, the error dynamics is asymptotically stable and the equivalent injection term  $v_{eq}$  can be approximated to the disturbance  $a_p$  after all errors of the sliding mode observer converge to near zero. The equivalent injection term was derived by multiplying the first order delay function with the time constant and injection term as follows.

$$a_p \approx v_{eq} = \frac{1}{\tau_v s + 1} v. \quad (16)$$

where  $\tau_v$  is the time constant for the derivation of the equivalent injection term and  $s$  is the complex variable of the Laplace transform. The magnitude of the estimated disturbance's change rate was used to compute the time constant for the adaptive weighting function. The next section describes the performance evaluation results obtained using the commercial software CarMaker.

### III. PERFORMANCE EVALUATION

Figure 4 shows a detailed model schematic for performance evaluation of the MPC algorithm with the proposed adaptive weighting function.

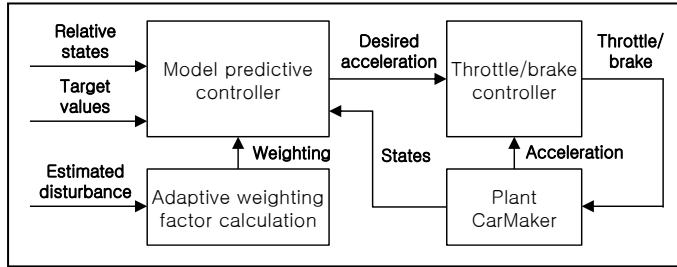


Figure 4. Model schematics of performance evaluations

The commercial software CarMaker was used for the performance evaluation of the proposed algorithm described in Fig. 4. To obtain the clearance and relative velocity between the preceding and subject vehicles, the velocity profile obtained by HILS (Human in the Loop system) was used as the longitudinal velocity of the preceding vehicle for performance evaluation. The HILS consists of the software (CarMaker) and hardware parts that are equipped with steering wheels and pedals for acceleration/braking to receive the driver's inputs.

In the velocity profile, there are two deceleration regions having  $-3 \text{ m/s}^2$  and  $-4 \text{ m/s}^2$  as a deceleration value. The throttle and brake inputs for desired acceleration tracking were controlled using a PID (proportional-integral-derivative) controller. The evaluation results from the adaptive weighted prediction-based MPC were compared to the ones from the MPC without the adaptive weighted prediction component (i.e., constant prediction). Figs. 5-10 show the evaluation results of the vehicle dynamic behaviors (velocity, clearance), control

input (desired acceleration), estimated disturbance, error states of clearance and velocity, and time constant.

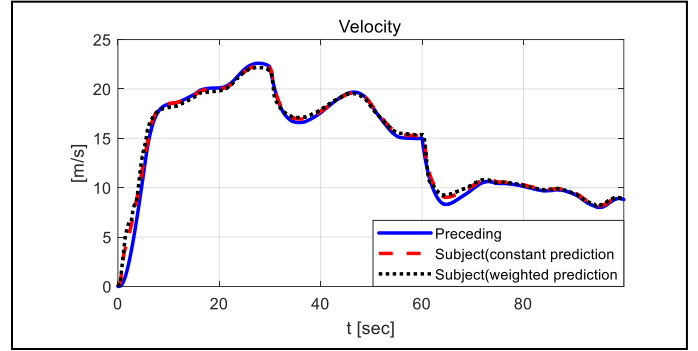


Figure 5. Results: velocity of the preceding and subject vehicles

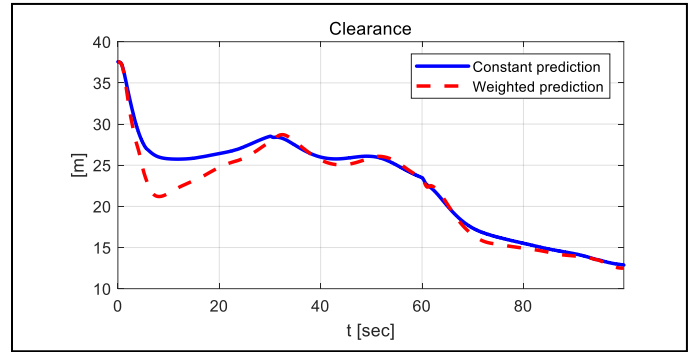


Figure 6. Results: clearance

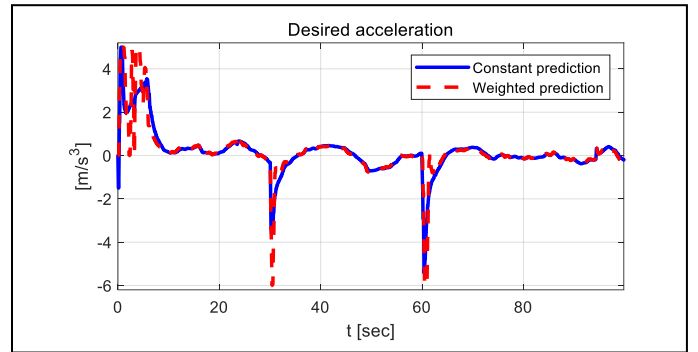


Figure 7. Results: desired acceleration

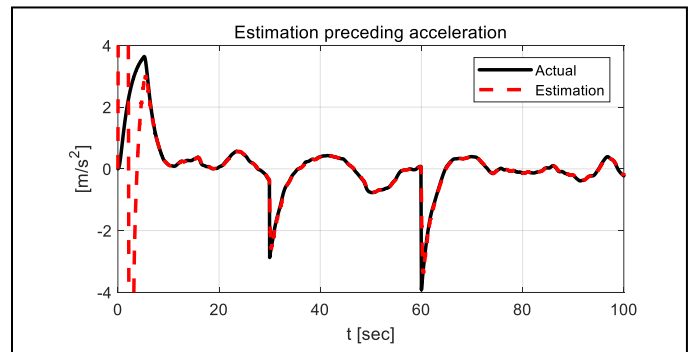
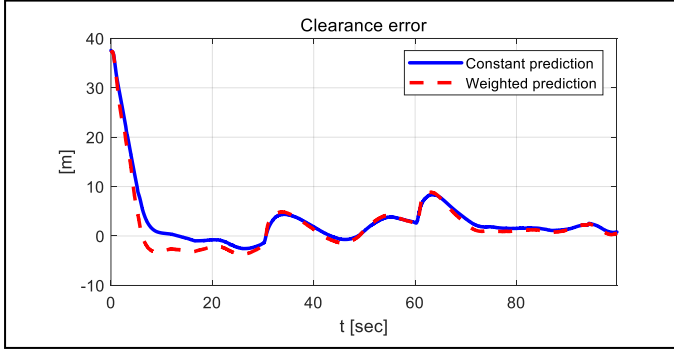
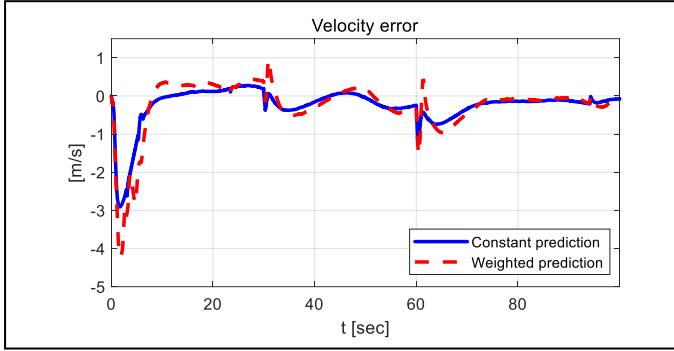


Figure 8. Results: estimated disturbance (longitudinal acceleration,  $a_p$ )



(a) Clearance error



(b) Velocity error

Figure 9. Results: error states (clearance and velocity)

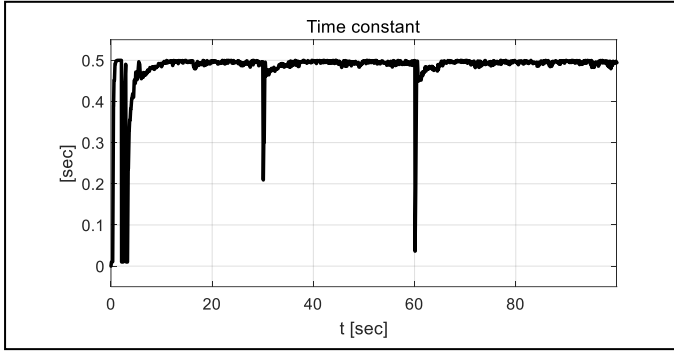


Figure 10. Results: time constant

As shown in Figs. 5 and 6, the subject vehicle can track the desired velocity (preceding vehicle velocity) reasonably within the clearance of 10 m - 40 m for all cases (both constant prediction and adaptive weighted prediction). In Fig. 7, the weighted prediction-based MPC shows a larger desired acceleration than the MPC with constant prediction. Figure 8 presents that the estimated disturbance (longitudinal acceleration of the preceding vehicle) obtained using the designed sliding mode observer falls within a small range of error (i.e.,  $\pm 0.05$  between actual and estimated signals) except for the regions of convergence (0-5 sec) and two decelerating edges (around 30 sec and 60 sec). This result means that the equivalent injection term can represent the actual longitudinal

acceleration of the preceding vehicle. Figure 9 shows the clearance and velocity errors. In the case of the weighted prediction-based MPC, the clearance error approached zero faster compared to the case of constant prediction-based MPC (0-5 sec) due to a larger deceleration value. After 30 sec, there is no significant difference in clearance between the two cases. The magnitude of velocity errors in the case of weighted prediction-based MPC is a bit larger than that with a constant prediction-based MPC around 30 sec and 60 sec due to its relatively higher acceleration. Therefore, the weighting function needs to be improved by considering the error states, which enables to ameliorate the control performance in any driving conditions. The time constant that was computed using the proposed rule in Eqs. (7)-(9) is shown in Fig. 10. When the preceding vehicle decelerates at 30 sec and 60 sec, the time constant value was decreased due to the sudden change of the estimated longitudinal acceleration (i.e., increase in the magnitude of its change rate). The decrease of the time constant value results in reducing the weighting function value and thus increasing the desired longitudinal acceleration of the subject vehicle as the prediction step increases.

#### IV. CONCLUSION

This study proposes the MPC algorithm with adaptive weighted prediction and disturbance estimation for longitudinal autonomous driving. The longitudinal acceleration of the preceding vehicle considered as a disturbance was estimated using a sliding mode observer. The estimated disturbance was utilized to compute the time constant for the weighting function designed by a decreasing exponential function. To reduce the negative effect of uncertainties on the control performance of the MPC, the weighting function was designed to adjust the individual cost value on the predicted states in the total cost function. The performance evaluation was conducted using CarMaker software. Evaluation results indicate that the adaptive weighted prediction-based MPC has a good capability for target tracking control with a large desired acceleration when the disturbance is changed dramatically. However, decreasing cost values for the predicted states in the MPC process does not always have a positive influence on control performance under unexpected and unpredictable disturbances. Therefore, the development of an adaptation algorithm for weighting values in a cost function of the MPC is considered as future work. Since some parameters such as minimum and maximum time constant values were determined for only one autonomous driving scenario, the control performance can be improved through parameter optimization or parameter adaptation algorithms under various scenarios. Therefore, this theme can be considered as a topic for future work. The methodology development for online self-tuning of the parameters using states and inputs can be another future extension since it can contribute to the enhancement of control robustness by dealing with various driving conditions. Finally, parameter optimization of finite-time stability conditions can be an extended work to enhance the estimation performance of a sliding mode observer. From successful findings from this study, it is expected that the proposed control algorithm can be extensively applicable to multivariable model predictive controllers as an adaptation approach.

## ACKNOWLEDGMENT

N/A

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