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COMPARISON OF HIGH-ORDER SCHEMES USING THE CARTESIAN CUT-STENCIL METHOD FOR NUMERICAL SOLUTION OF PDES

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ABSTRACT

The concept of a “cut-stencil” combined with the finite difference method has been successfully applied to numerically solve partial differential equations on arbitrary domains with complex irregular and curved boundaries. Since a Cartesian mesh can be used to discretize a complex region, it has been used to develop a new pure finite difference formulation. Most of the published research with this method has focused on achieving second-order accurate solutions of the linear convection-diffusion equation and the streamfunction-vorticity equations of 2D incompressible fluid flow.

Many researchers have used the Cartesian mesh approach with the finite volume method, and all have to deal with the issue of cut cells, i.e., those rectangular cells that are cut when they intersect with the boundary of the flow domain. This leads to complicated cell connectivity, volume and face information. In the current study, our approach is based on the 5-point stencil (in 2D) attached to each node in the mesh instead of the cells surrounding each node. The mesh can be constructed so that the stencils are uniform for nodes in the domain interior, but arms of some stencils may be cut for nodes near a boundary. For this reason, we refer to this approach as the “Cartesian cut-stencil” method. In this case, standard finite difference formulae can be applied at interior nodes, but the non-uniform stencils at near-boundary nodes render the finite difference approach inapplicable at these nodes. A similar situation arises if the mesh is locally clustered as, for example, in the case of a boundary layer mesh. The Cartesian cut-stencil method overcomes this limitation by mapping non-uniform physical stencils onto a uniform computational stencil.

In this research we discuss a novel high-order accurate finite difference algorithm for the numerical simulation of 2D steady flows, based on the Cartesian cut-stencil concept. Several strategies for higher-order solutions have been proposed, including compact Hermitian operators, non-compact wide stencil schemes and Richardson extrapolation. However, we have recently designed several relatively simple alternative high-order schemes, including an innovative compact scheme which relies on the placement of ghost nodes inside the stencil, allowing very high-order schemes without degrading the accuracy at the domain boundaries. With these new schemes, fourth-order accuracy can be preserved at all nodes in the mesh without requiring special formulae for stencils that are adjacent to a boundary. Several examples will be used to illustrate the accuracy that is achieved from these various 4^{th}-order schemes.